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# Scale invariance and universality of economic fluctuations

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**Abstract**

In recent years, physicists have begun to apply concepts and methods of statistical physics to study economic problems, and the neologism "econophysics" is increasingly used to refer to this work. Much recent work is focused on understanding the statistical properties of time series. One reason for this interest is that economic systems are examples of complex interacting systems for which a huge amount of data exist, and it is possible that economic time series viewed from a different perspective might yield new results. This manuscript is a brief summary of a talk that was designed to address the question of whether two of the pillars of the field of phase transitions and critical phenomena – scale invariance and universality – can be useful in guiding research on economics. We shall see that while scale invariance has been tested for many years, universality is relatively less frequently discussed. This article reviews the results of two recent studies – (i) *The probability distribution of stock price fluctuations*: Stock price fluctuations occur in all magnitudes, in analogy to earthquakes – from tiny fluctuations to drastic events, such as market crashes. The distribution of price fluctuations decays with a power-law tail well outside the Lévy stable regime and describes fluctuations that differ in size by as much as eight orders of magnitude. (ii) *Quantifying business firm fluctuations*: We analyze the Computstat database comprising all publicly traded United States manufacturing companies within the years 1974–1993. We find that the distributions of growth rates is different for different bins of firm size, with a width that varies inversely with a power of firm size. Similar variation is found for other complex organizations, including country size, university research budget size, and size of species of bird populations. 

## 1. Introduction

We organize our presentation around three questions: (i) What *is* the question? (ii) Why do we care about this question? and (iii) What do we actually do?

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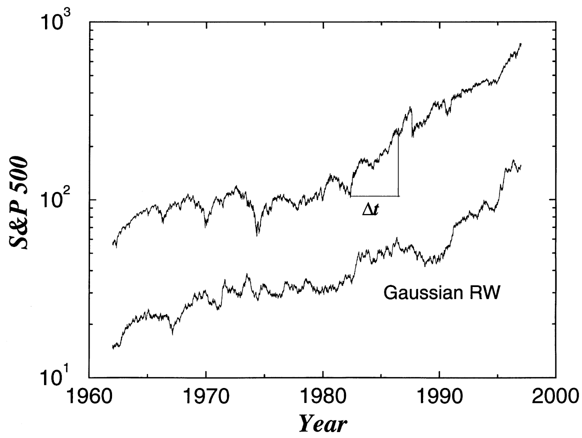


Fig. 1. Comparison of the time evolution of the S& P 500 for the 35-year period 1962–1996 (top line) and a biased Gaussian random walk (bottom line). The random walk has the same bias as the S& P 500 – approximately 7% per year for the period considered. The sharp drop seen in the S& P 500 in 1987 is the market crash of October 19. Courtesy of P. Gopikrishnan.

### *1.1. What is the question?*

To a physicist, the question is very basic: *How can we quantify economic fluctuations*? Everything in the economy fluctuates. Consider a single "fruit fly", one *drosophila* in this field of economics – the S& P 500 index, which is a weighted average of the 500 largest US business firms. When graphed against time on the $x$-axis, one sees substantial price fluctuations over time, even if price changes are shown logarithmically on the $y$-axis (Fig. 1). Note, in particular, the very large "fluctuation" that took place on 19 October 1987 (Black Monday), when business firms all over the world lost from 20 to 50 percent of their market value. Black Monday occurred simultaneously all over the world – a first analogy with classic critical phenomena, since obviously at that moment every stock in the world clearly depended on every other stock.

Fig. 1 compares this experimental data to a plot generated by a simple biased random walk, i.e., a pattern created by the sequential flipping of a biased coin – if the coin is heads, the walker moves up; if tails, the walker moves down. We notice that, although this simple theoretical model – which was first developed by Bachelier [1] at the end of the last century – does bear a striking resemblance to the empirical curve, there are no "Black Mondays" in the theoretical model. The reason for this is intuitively obvious: the probability of a tossing $N$ tails in a row – which would be necessary to cause the walker to make $N$ steps in the down direction – is $(\frac{1}{2})^N = \exp(-N \log 2)$, which decreases exponentially with $N$. Hence a major "crash" in this theoretical model is "exponentially rare" – i.e., virtually impossible.

## 1.2. Why do we care about this question?

### 1.2.1. Scientific reasons

A first scientific reason to care about the fluctuating economic system is that the topic allows us to test ideas of complex systems on thoroughly documented databases; literally every economic transaction is recorded somewhere and, in principle, recorded very accurately. That accuracy is not present in the physical measurements that constitute most of the databases we physicists work with, and that completeness is also not present in our usual databases (since we do not have an infinite number of graduate students to take all possible measurements).

A second scientific reason to care about this topic is the ready possibility of interactions between certain unsolved problems in physics – such as turbulence – and unsolved problems in economics. Turbulence bears a remarkable resemblance to economics. A financial news source can report that yesterday was a "turbulent day" on Wall Street and, qualitatively, we understand the phenomenon that is being described – but, as physicists, we can go much further than this general sense of what is taking place. Indeed, the stock market and turbulence have been analyzed in parallel by a number of researchers in recent years [2–4].

If we take a glass of water and stir it, energy on a big scale (our induced stirring) is dissipated on smaller and smaller scales (the ripples that go out from our induced stirring). In an economic system, induced energy takes the form of new information. If there is no new information, there is no motivation for action. In a news blackout, all trading stops. For this reason, Wall Street keeps track of *all* news, utilizing special "live wire" news sources; they pay large sums of money in order to hear every piece of news – correct or incorrect – the instant it happens or is reported to have happened.

Recently, I received an e-mail from a 21-year-old relative who works on Wall Street saying there was an earthquake in Mexico City. I immediately sent an e-mail to a colleague in Mexico City asking if they had experienced an earthquake. He instantly answered yes, but that it had only begun 30 s ago – and how did I know? For me, that is vivid proof that Wall Street is poised for almost instantaneous action whenever news happens.

News on Wall Street, like injected energy in turbulence, is dissipated at smaller and smaller scales. One big firm might take some action at the news of the Mexico City earthquake, and another smaller firm might take a different action, and a retiree or day-trader still another.

### 1.2.2. Practical reasons

The overriding practical reason to care about this question is that this fluctuating economic system affects everyone in the world. Since in economics "everything depends on everything else", a Black Monday is a serious event for all of humanity.

An investment firm on Wall Street recently tried using a variant of the simple biased random-walk model. Eventually, and not surprisingly, the law of statistics caught up with them: a huge fluctuation occurred and the firm was unable to cover its debts. Like all similar firms on Wall Street, it had invested more money than it actually had (such firms typically invest 20 times as much). What happened then? Did the firm's creditors give them concrete overshoes and dump them in the harbor – the usual Hollywood solution for gamblers who cannot cover their debts? No! Other investment firms on Wall Street reached into their pockets and pulled out billions in order to bail them out.

This was not altruism run amok; it was a conscious action taken in self-interest. The other firms knew that in the economy everything is affected by everything else and that, if a major firm failed, they also could be powerfully affected. Investors would become frightened enough to withdraw their money, not just from the failed firm (if they could get any), but also from firms similar to the one that failed. Typically, firms have readily available only a small fraction of the total funds they control, so a large increase in withdrawals usually has a devastating effect.

Another way of describing this practical reason to care about the fluctuating economic system is to say that we want to be able to predict, and perhaps prevent, "economic earthquakes". If Wall Street investment firms think they can manage huge chunks of the economy using a simple biased random-walk model that neglects big fluctuations, then it seems we face a new kind of brinksmanship to replace the one we grew to know so well during the Cold War. This is not idle speculation. There was a regional economic earthquake recently in Indonesia – a collapse in the economy caused, among other things, a rice shortage, and people actually starved. Investors in that economy were affected because they lost their money, and a large number of non-investors were affected because they no longer had food. Instances like this one make it clear that the economy has become crucial to everyone's well being, not just the well being of stockholders.

## 2. What do we actually do?

### 2.1. *Quantifying stock price fluctuatuations*

When we physicists look at the fluctuating economic system, the first thing we want to do is quantify the fluctuations [5–8]. One way we can do this is to measure the change $G$ in the value of the stock price – first now, and then again after some fixed time interval $\Delta t$ (say 1 day). Of course, $G$ will change with time. On good days – when the market goes up – $G$ will be positive, and on bad days $G$ will be negative. What can we physicists do with this information? We can measure the time-correlation function, and if these fluctuations are indeed a conventional kind of critical phenomena, the correlation should be a power law. A number of researchers have already done this – we were not the first – and discovered that it is *not* a power law [9,10]. In fact, a log-linear plot of the correlation function is a straight line, and the fall-off – the characteristic value of the exponential that gives rise to that straight line – is four minutes.

What about the absolute value of $G$? If we make a log–log plot of the auto-correlation function of the absolute value of $G$, we find a region of roughly two decades where the data are approximately linear, indicating that there are some kind of long-range power-law correlations in that quantity [9,10]. Of course, you cannot make money knowing only the absolute value of $G$ – you have to know whether the quantity is going up or down. An intriguing fact is that when we study the long-range power-law correlations in the absolute values of $G$ – using power-spectrum methods or detrended fluctuation analysis – we see an interesting crossover at approximately one day [10]. The reason for this crossover is unknown. Perhaps one of the students attending this conference will help us understand the reason?

In addition to analyzing the data in the order they are actually recorded, we can simply "dump the data on the floor", and then retrieve the data points and bin them according to value. That is exactly what Benoit Mandelbrot did – as well as many others before and after him. Mandelbrot did it with three sets of data – two sets of daily returns and one set of monthly returns – for cotton prices [5]. When he constructed a cumulative histogram, i.e., the number of times a cotton price fluctuated *more* than a certain amount $G$ on the $x$-axis, he got linear behavior on log–log paper. He also discovered what today is called "universality" – the same slope, 1.7, in each of the three data sets. This was both "scaling" and "universality" in cotton price fluctuations. The fact that the data were linear on log–log paper replaced the Bachelier theory of a simple biased random walk [1] with another distribution, the Lévy distribution. Since that time, many have worked with Mandelbrot's findings and have given them a good deal of credence, but – as we will soon see – that Lévy distribution does *not* hold for the data we are examining.

Each of Mandelbrot's analyses had only $\approx 10^3$ data points, and the shortest time interval studied was 1 day. Shorter time intervals are helpful for a detailed study of fluctuations. Mantegna studied these fluctuations with a different database, the S& P 500 index, in which the fluctuations are measured down to 15-s time periods $\Delta t$ and the data points number not $\approx 10^3$ but $\approx 10^6$ [2–4,9]. Mantegna's histogram of value fluctuations in the S& P 500, which are graphed logarithmically on the $y$-axis, do not show an inverted parabola ($-x^2$). Rather, the data show fluctuations far bigger than those predicted by a Gaussian, i.e., a Gaussian does not have any "Black Mondays". His histogram also shows, when we look at the center of the data, a Lévy distribution. The fit is remarkably good out to about five standard deviations, supporting Mandelbrot's results. When we probe the wings, we find that the data are orders of magnitude below the predictions of a Lévy distribution. If the empirical data did follow a Lévy distribution, "Black Mondays" would occur much more frequently. This distribution, which is a truncation of the Lévy form, is called a truncated Lévy flight [11–13].

What about the "universality" of this distribution of price changes? Johannes Skjeltorp did an analysis similar to that of Mantegna's, but on the Norwegian economy [14]. Even though Norway's economy is only 5% as large as the USA, and is extremely specialized because of its dependence on oil revenue, Skjeltorp finds the same truncated Lévy distribution as was found in the S& P 500.

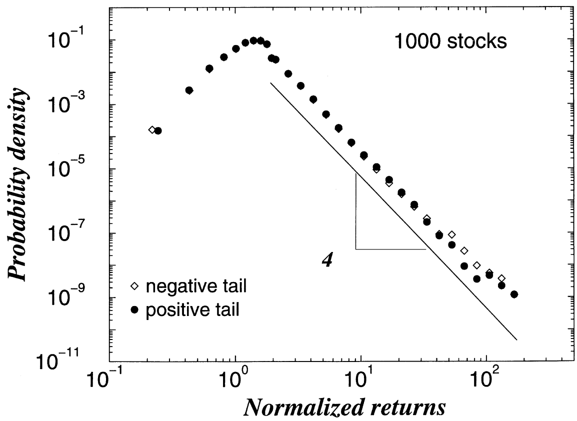


Fig. 2. Probability density function of the normalized returns of the 1000 largest companies in the TAQ database for the 2-year period 1994–1995. A power-law fit in the region $2 \leqslant x \leqslant 80$ yields values of the slope $1 + \alpha = 4.10 \pm 0.03$ for the positive tail and $1 + \alpha = 3.84 \pm 0.12$ for the negative tail, clearly outside the Lévy stable domain $1 < 1 + \alpha < 3$. The fall off of the distribution for small values of returns arise from the discreteness in stock prices, which are set in units of fractions of USD, usually $\frac{1}{8}$, $\frac{1}{16}$, or $\frac{1}{32}$. Courtesy of P. Gopikrishnan.

Just as Mantegna increased the number of data points from $\approx 10^3$ to $\approx 10^6$ in his analysis, Gopikrishnan and collaborators increased the number from $\approx 10^6$ to $\approx 10^8$ by analyzing the data for every transaction made in the entire 2-year period 1994–1995 [15–17]. He found that the distribution $P(G)$ is approximately linear on log–log paper (Fig. 2). When he averaged over every individual stock, he convincingly showed that for individual stocks the histogram of price fluctuations is very close to an inverse quartic power-law. The data fall on the same straight line all the way out to 100 standard deviations – i.e., events that are $\approx 10^{-8}$ as common as everyday events, since two decades on the abscissa is eight decades on the ordinate if the exponent is $-4$ [15–18]. This means that extremely rare events seem to be following the same empirical law as common everyday events.

Unlike the apparent differing behaviors in the S& P 500 – a region of Lévy behavior and a region of truncation – in Gopikrishnan's histogram of individual stocks there is no region of Lévy behavior at all. Gopikrishnan repeated Mantegna's work using a longer time frame and confirmed that there is a small region of Lévy behavior, with truncation by a power law. Again, earthquakes that are $\approx 10^{-8}$ as common as everyday shocks follow the same plot.

The Gutenburg–Richter law for the frequency of earthquakes is of the same character. Large earthquakes fall on the same straight line as small earthquakes, meaning that extremely rare events seem to be following the same empirical law as common everyday events. This empirical fact suggests that if we can build models and develop some understanding of everyday "small earthquakes", then this understanding will help comprehend large earthquakes – we do not need a separate theory for the large earthquakes.

Similarly, perhaps statistical physicists can develop some understanding of everyday "small" stock price fluctuations, we will be able to use this understanding to comprehend large price fluctuations – we will not need a separate theory (as some believe we do) for the large "economic earthquakes".

## 2.2. Quantifying business firm fluctuations

The theory of firms in economics is not unlike the theory of the Ising model of spin glasses in physics. The set of "all spins" here is the set of all publicly traded firms. The classic approach in economics is to divide that "set of all spins" into "sectors": manufacturing, food, computers, drugs (the legal kind), automotive, and so forth. The interactions within sectors are treated exactly, but the interactions between sectors are treated using a mean-field theory or are neglected altogether. The classic approach in physics is the same thing as our so-called effective-field theories of magnetism [19]. However, in physics we have found that such effective-field theories miss the essential physics under conditions of criticality [20].

So, in order to avoid the same problem in economic systems, we analyzed all the firms together – rather than breaking them into sectors. That each firm has some dependence on all other firms is more realistic for intuitive reasons. If General Motors goes down because of the public disclosure of an unsafe design in some of its vehicles, then Ford may go up as customers, concerned about safety, purchase that manufacturer's cars instead. But then Ford may need to expand their work force to make the increased number of cars, and that larger number of workers almost certainly will increase the noontime business at the McDonald's across the street from the plant. So we have an interaction that negatively correlates two firms in the same sector, and one that positively correlates two firms in two different sectors.

This spin-glass picture of the economy is still new and seems very promising, but we have not yet discovered exactly how to work with it. There are $10^4$ firms and so $10^8$ interactions – interactions that are sometimes positive, sometimes negative, some are long-range, and some even change with time, and we do not even know how to assign the interaction energy $J$.

So we find ourselves once again making histograms. Specifically, Stanley and collaborators [21,22] made a histogram that shows how many times a firm grows by a given amount – e.g., the sales this year divided by the sales last year – places the histogram into a family of histograms, depending on the size of the firm. As we would expect, a large firm cannot grow or shrink very much in a given year. General Motors could not grow by a factor of 10 in a single year. Big firms have narrow histograms. Small firms, on the other hand, have very wide histograms (Fig. 3). The width, or standard deviation, of a histogram is a decreasing function of the size of the firm. This has been known qualitatively for some time, but apparently no one had actually tried to quantify how that decrease depends on the size of the firm. When we make that plot we find that the standard deviation is a function of the size of the firm, and the data are approximately linear on log–log paper – suggesting that that deviation is a power

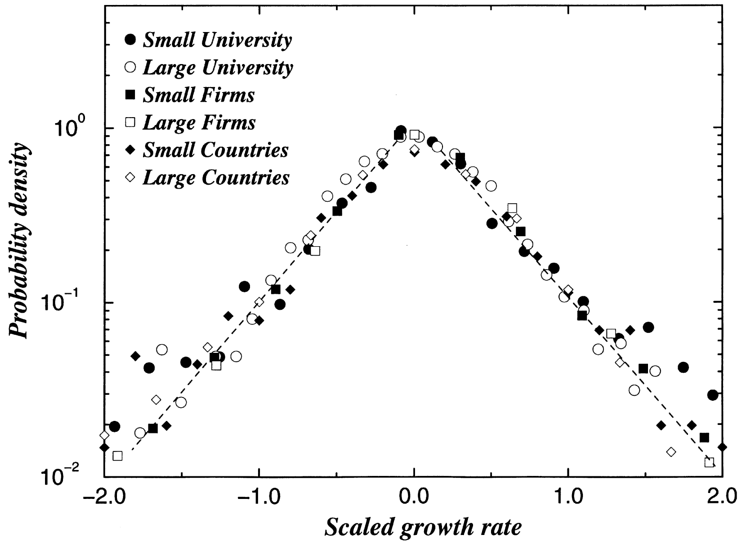


Fig. 3. Distribution of scaled annual growth rates for different organizations: R& D expenditures of US universities, sales of firms, and GDP of countries. The data collapse onto a single tent-shaped curve suggesting that the scaled distributions have the same functional form. Courtesy of V. Plerou.

law. The slope of that power law is on the order of 0.2. This number also seems to be quite universal, e.g., if you measure firm size by number of employees (rather than by annual sales total) the slope is the same. It is therefore possible to collapse all these tent-shaped distributions onto the same scaling curve by using appropriate scaled variables.

What about universality? Takayasu tested the universality hypothesis by looking at data for other countries and data for individual sectors of the economy [23]. Jeffrey Sachs of Harvard University suggested that we look at GDP data of countries. For some time, one has known that the "economies" of many large business firms are the size of those of small countries, but we are just now becoming aware that business firms might also have similar organizational structures to countries. Canning, Lee, and coworkers found that indeed the identical tent-shaped distribution was found for the growing and shrinking of the economies of countries and with the identical slope ($\approx\frac{1}{6}$) [24,25]. So, in this sense, countries and business firms are identical.

Recently Plerou, Amaral, and coworkers found parallels between business firms and funded scientific research in universities [26]. We researchers must sell our ideas to the funding institutions, so we are the business firm and the funding institution is the customer. The growth and shrinking of research funding, when analyzed, exhibits the same tent-shaped distribution and, when size-dependence is scaled out, everything can be plotted on the same curve (Fig. 3).

This has even been applied to non-human organizations. Each species of bird is like a firm in that it grows or shrinks from year to year. Keitt found that data on the dynamics of North American breeding-bird populations demonstrate the same tent-shaped distribution [27].

## 3. Summary

In recent years, physicists have started applying concepts and methods of statistical physics to study economic problems, and the neologism "econophysics" was introduced in 1995 to refer to this work [28]. In this presentation, we have reviewed two recently uncovered empirical results that appear to be "universal" in that they are independent of the details of the economic system studied:

(1) The Lévy distribution is not valid for stock price fluctuations. It *appears* to be valid for certain stock *averages*, if the tails are ignored, but it is *never* valid for stock price fluctuations of *individual firms*, even when the tails are ignored.

(2) The growth and shrinking of complex organizations appears to follow a universal law that is immensely robust. This universal law demonstrates its validity for a wide range of organization types – from business firms to bird populations.

A possible key to understanding the first empirical law is to consider stock price fluctuations as analogous to diffusion. In diffusion, a particle changes its direction whenever there is a molecular collision. In stock price fluctuations, the change takes place whenever there is a transaction. If we carry though the steps of classic diffusion, we find predictions that do not agree with the stock market – but if we *generalize* classic diffusion to anomalous diffusion (a test particle in Yellowstone Park, bouncing around among the bubbling springs and geysers) we do get the predictions of the stock market [29].

Regarding the tent-shape distributions observed for the growth of organizations and the corresponding power laws, we have recently developed a simple model that reproduces those empirical findings [30]. The model dynamically builds a diversified, multi-unit structure, reproducing the fact that the typical organization passes through a series of changes in structure, growing from a single-unit to a multi-unit organization. Our simple model rests on a small number of assumptions: (i) organizations tend to grow into multiple divisions once they achieve a certain size, (ii) there is a broad distribution of characteristic scales in the system, and (iii) growth rates of different divisions are independent of one another.

The model provides some insight into the processes by which organizations grow and leads to a number of conclusions. Namely, it suggests the deviations in the empirical data from predictions of a simple random multiplicative process may be explained (i) by the diversification of firms, i.e., firms are made up of interacting subunits; and (ii) by the fact that different industries have different underlying scales, i.e., there is a broad distribution of minimum scales for the survival of a unit (for example, a car manufacturer must be much larger than a software firm).

Moroever, the model suggests a possible explanation for the common occurrence of power-law distributions in complex systems. Namey, that the empirically observed power-law scaling does not require a critical state of the system, but may arise from an interplay between random multiplicative growth and the complex structure of the units composing the system.

We close with the caveat that we have not discussed many other interesting results in economics. One of the most promising relates to the possibility that correlations between different stocks can provide useful information and possible interesting new "laws of economics" (see, e.g., Refs. [31,32] and references cited therein).

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