ELSEVIER

Physica A 302 (2001) 126–137

PHYSICA A

www.elsevier.com/locate/physa

Quantifying economic fluctuations

H. Eugene Stanley[a,*], Luis A. Nunes Amaral[a], Xavier Gabaix[b], Parameswaran Gopikrishnan[a], Vasiliki Plerou[a]

[a] *Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA*
[b] *Department of Economics, Massachusetts Institute of Technology, Cambridge, MA 02142, USA*

Dedicated to Professor Dr. H.A. Weidenmüller on the occasion of his 65th birthday

Abstract

This manuscript is a brief summary of a talk designed to address the question of whether two of the pillars of the field of phase transitions and critical phenomena—scale invariance and universality—can be useful in guiding research on interpreting empirical data on economic fluctuations. Using this conceptual framework as a guide, we empirically quantify the relation between trading activity—measured by the number of transactions N—and the price change $G(t)$ for a given stock, over a time interval $[t, t+\Delta t]$. We relate the time-dependent standard deviation of price changes—volatility—to two microscopic quantities: the number of transactions $N(t)$ in Δt and the variance $W^2(t)$ of the price changes for all transactions in Δt. We find that the long-ranged volatility correlations are largely due to those of N. We then argue that the tail-exponent of the distribution of N is insufficient to account for the tail-exponent of $P\{G > x\}$. Since N and W display only weak inter-dependency, our results show that the fat tails of the distribution $P\{G > x\}$ arises from W. Finally, we review recent work on quantifying collective behavior among stocks by applying the conceptual framework of random matrix theory (RMT). RMT makes predictions for "universal" properties that do not depend on the interactions between the elements comprising the system, and deviations from RMT provide clues regarding system-specific properties. We compare the statistics of the cross-correlation matrix C—whose elements C_{ij} are the correlation coefficients of price fluctuations of stock i and j—against a random matrix having the same symmetry properties. It is found that RMT methods can distinguish random and non-random parts of C. The non-random part of C which deviates from RMT results, provides information regarding genuine collective behavior among stocks. We also discuss results that are reminiscent of phase transitions in spin systems, where the divergent behavior of the response function at the critical point (zero magnetic field) leads to large fluctuations, and we discuss a curious "symmetry breaking", a feature qualitatively identical to the behavior of the probability density of the magnetization for fixed values of the inverse temperature.

* Corresponding author. Tel.: +1-617-353-2617; fax: +1-617-353-3783.
E-mail address: hes@bu.edu (H.E. Stanley).

0378-4371/01/$ - see front matter
PII: S0378-4371(01)00504-0

Keywords: Econophysics; Random matrix theory; Volatility; Lévy distribution; Economics; Firm growth; Gross domestic product (GDP)

1. Introduction

In recent years, physicists have started applying concepts and methods of statistical physics to study economic problems. The word "Econophysics" is sometimes used to refer to this work. Much recent work is focused on understanding the statistical properties of financial time series. One reason for this interest is that financial markets are examples of complex interacting systems for which huge amounts of data exist and it is possible that financial time series viewed from a different perspective might yield new results. This article reviews the results of several recent studies, with emphasis on studies carried out by the authors.

(i) *The probability distribution of stock price fluctuations.* Stock price fluctuations occur in all magnitudes, in analogy to earthquakes—from tiny fluctuations to drastic events, such as market crashes. The distribution of price fluctuations decays with a power-law tail well outside the Lévy stable regime and describes fluctuations that differ by as much as 8 orders of magnitude. In addition, this distribution preserves its functional form for fluctuations on time scales that differ by 3 orders of magnitude, from 1 min up to approximately 10 days.

(ii) *Correlations in financial time series.* While price fluctuations themselves have rapidly decaying correlations, the magnitude of fluctuations measured by either the absolute value or the square of the price fluctuations has correlations that decay as a power-law and persist for several months.

(iii) *Correlations among different companies.* The third result bears on the application of random matrix theory to understand the correlations among price fluctuations of any two different stocks. From a study of the eigenvalue statistics of the cross-correlation matrix constructed from price fluctuations of the leading 1000 stocks, we find that the largest $\approx 5\%$ of the eigenvalues and the corresponding eigenvectors show systematic deviations from the predictions for a random matrix, whereas the rest of the eigenvalues conform to random matrix behavior—suggesting that these 5% of the eigenvalues contain system-specific information about correlated time evolution of different companies.

(iv) *Similarities with critical point phenomena.* We also discuss results that are reminiscent of phase transitions in spin systems, where the divergent behavior of the response function at the critical point (zero magnetic field) leads to large fluctuations. In particular, we discuss a curious "symmetry breaking" for values of Σ above a certain threshold value Σ_c; here Σ is defined to be the local first moment of the probability distribution of demand Ω—the difference between the number of shares traded in buyer-initiated and seller-initiated trades. This feature is qualitatively identical to the behavior of the probability density of the magnetization for fixed values of the inverse temperature.

2. Price fluctuations and market activity

Empirical evidence has been mounting to support the intriguing possibility that a number of systems arising in disciplines as diverse as physics, biology, ecology, and economics may have certain quantitative features that are intriguingly similar. These properties can be conveniently grouped under the headings of scale invariance and universality [1]. Scale invariance refers to a hierarchical organization that results in power-law behavior over a wide range of values of some control parameter—such as species size, heartbeat interval, or firm size—and the exponent of this power-law is a number characterizing the system. By universality, we mean a tendency for the set of exponents found for diverse systems to partition themselves into distinct "universality classes", with the property that all systems falling into the same universality class have the same exponent—suggesting that there are features in common among the underlying microscopic mechanisms responsible for the observed scale invariant behavior.

Researchers have found new and surprising results by applying concepts and methods of scale invariance and universality to the economy. The economy is perhaps the most complex of all complex systems [2–8]. A very small piece of "bad news" in a remote market may trigger a very large response in financial indices all over the globe. The societal impact of such economic fluctuations can be devastating. Privately, economists will confirm that the probability of such an "economic earthquake"—a sudden and disastrous "phase transition" from the present healthy state of our economy to a new state of a completely devastated economy—is not entirely negligible. A celebrated example of the societal devastation caused by economic earthquakes is the collapse of the German economy following World War I, which directly contributed to the rise of Hitler. Another example is the recent "devaluation" in Indonesia that has contributed to the starvation of Indonesia's poor.

In the case of economics, virtually every economic transaction has been recorded—somewhere. The challenge is to obtain the needed data and to analyze them in such a way as to reveal the underlying principles. Remarkably, one finds that if one makes a histogram of price changes for any stock (the analog of the Gutenberg–Richter histogram of earthquake magnitude [9,10]) this histogram is very close to a power-law [11,12]. This discovery suggests that large shocks are related in a scale invariant fashion to smaller, commonplace, economic fluctuations—i.e., large shocks and everyday economic fluctuations are basically different manifestations of the same phenomenon. The greatest societal impact occurs when "the big one" occurs, whether it be a geophysical earthquake or an economic earthquake. Hence scaling concepts make it possible for scientists to understand these rare but catastrophic events through appropriately designed research focused on everyday phenomena.

Stock price fluctuations display distinctive statistical features that are in stark contrast to those of a simple random walk ("diffusion") model. Consider price change

$$G(t) \equiv \ln S(t + \Delta t) - \ln S(t)\,, \tag{1}$$

defined as the change in the logarithm of price $S(t)$ over an interval Δt. Empirical work shows that the distribution function $P_G\{G > x\}$ has tails that decay as a power-law

$$P_G\{G > x\} \sim x^{-\alpha}\,, \tag{2}$$

with α larger than the upper bound ($\alpha = 2$) for Lévy stable distributions [11–13]. In particular, studies on the largest 1000 US-stocks [12] and 30 German stocks [11] show mean values of $\alpha \approx 3$ on time scales $\Delta t \leqslant 1$ day. Secondly, it is found that although the process $G(t)$ has a rapidly decaying autocorrelation function $\langle G(t)G(t+\tau)\rangle$, which at time scales $\tau < 30$ min, shows significant anti-correlations (bid–ask bounce) for individual stocks, but cease to be statistically significant for larger time scales. Higher-order two-point correlation functions show quite a different behavior. For example, the autocorrelation function of the absolute value of price changes show long-range persistence

$$\langle |G(t)|\,|G(t+\tau)|\rangle \sim \tau^{-\mu}\,, \tag{3}$$

with $\mu \approx 0.3$ [14–16].

The problem of understanding the origin of these features is a challenging one [17,18]. This paper reviews recent work which focuses on a much more modest goal of trying to understand, starting from transactions, how these statistical features—fat-tailed distributions and long-ranged volatility correlations—originate. We shall show that the price changes, when conditioned on the volatility, have tails that are consistent with those of a Gaussian. In addition, we shall show that the long-ranged correlations in volatility arise from those of trading activity measured by the rate of occurrence of trades N. However, the distribution characteristics of trading activity implies that the fat tails of G cannot arise solely due to N. We relate the fat-tailed behavior of G to those of "transaction-time" volatility W which, roughly speaking, measures the impact of trades.

Let us start by examining the conventionally used "geometric"-variant of Bachelier's "classic diffusion" model. The rationale for this model arises from the central limit theorem by considering the price changes G in a time interval Δt as being the sum of several changes δp_i, each due to the ith transaction in that interval,

$$G \equiv \sum_{i=1}^{N} \delta p_i\,, \tag{4}$$

where N is the number of transactions (trades) in Δt. If $N \gg 1$, and δp_i have finite (constant) variance W^2, then one can apply the central limit theorem, whereby one would obtain the result that $P_G(G)$ is Gaussian with variance $\sigma^2 = W^2 N$, and therefore prices evolve with Gaussian increments. It is implicitly assumed in this description that N is almost constant, or more precisely, N has only narrow (standard deviation much smaller than the mean) Gaussian fluctuations around a mean value. Let us start by asking to what extent this is true.

Let us first quantify the statistics of N. We first analyze the distribution of N for 1000 stocks, and find that $P(N)$ decays as a power-law,

$$P_N\{N > x\} \sim x^{-\beta}\,, \tag{5}$$

with values of β around the average value $\beta = 3.4$ [19].

Since N behaves in a non-Gaussian manner, one can ask whether the exponent α for the distribution of price changes $P_G\{G > x\} \sim x^{-\alpha}$ arises from the exponent β for P_N. To address this problem, we must first quantify the relationship between G and N. Consider the conditional distribution $P_{G|N,W}(G|N,W)$ for given values of N and W. If we assume that the changes δp_i due to each transaction in Δt are *i.i.d.*, then the variance of $G(t)$ in that time interval will be W^2N. Thus, the width of the conditional distribution $P_{G|N,W}(G|N,W)$—probability density of G for given values of N and W—will be the standard deviation $W\sqrt{N}$, which measures the local volatility. If we next hypothesize that the *functional form* of $P_{G|N,W}(G|N,W)$ does not depend on the values of W or N, then we can express

$$P_{G|N,W}(G|N,W) = \frac{1}{W\sqrt{N}} f\left(\frac{G}{W\sqrt{N}}\right)\,, \tag{6}$$

where the function f has the same form for all values of W and N. [1] In other words, during periods of large $W\sqrt{N}$, the conditional distribution $P_{G|N,W}(G|N,W)$ will have large width.

We seek to quantify the functional form of the conditional distribution $P_{G|N,W}$. Under our hypothesis, determining the conditional distribution is tantamount to determining the functional form f, which is accomplished by considering a "scaled" variable

$$\varepsilon \equiv \frac{G}{W\sqrt{N}}\,, \tag{7}$$

which is free of the effects of fluctuating $W\sqrt{N}$. Our examination of the distribution $P_\varepsilon(\varepsilon)$ shows that it is consistent with Gaussian behavior [19]. Thus, the conditional distribution is consistent with the functional form [2]

$$P_{G|N,W}(G|N,W) \simeq \frac{1}{\sqrt{2\pi}W\sqrt{N}} \exp\left(\frac{-G^2}{2W^2N}\right)\,. \tag{8}$$

We are now in a position to relate the statistical properties of G and N. One can express the distribution of price changes P_G in terms of the conditional distribution $P_{G|N,W}(G|N,W)$ or, equivalently, in terms of f,

$$P_G(G) = \int \frac{1}{\xi} f\left(\frac{G}{W\sqrt{N} = \xi}\right) P_{W\sqrt{N}}(\xi)\,\mathrm{d}\xi\,, \tag{9}$$

[1] The hypothesis that the conditional distribution has the same form for all W and N might strike the reader as surprising since one expects the conditional distribution to be increasingly "closer" to a Gaussian for increasing N. Strictly speaking, if W and N are independent, then the hypothesis would be exact only for a stable distribution for δp_i such as a Gaussian (consistent with our findings later in the text).

[2] The $\simeq$ sign is used because although the tails of the conditional distribution are consistent with Gaussian, the central part is affected by discreteness of price changes in units of 1/16 or 1/32 of a dollar.

where $P_{W\sqrt{N}}$ denotes the probability density function of the variable $W\sqrt{N}$. Since f is consistent with Gaussian, it is clear that the fat tails in G must arise due to the mixing of the conditional distribution, averaged over all possible widths $W\sqrt{N}$.

Next, we examine how the statistics of W and N relate to the statistics of G. First, we examine the equal-time dependence of W and N and find that the equal-time correlation coefficient is small, suggesting only weak interdependence [19]. Therefore, the contribution of N to the distribution $P_{W\sqrt{N}}$ in Eq. (9) goes like the distribution of $\sqrt{N}$. We have already seen that the distribution $P_N\{N > x\} \sim x^{-\beta}$ with $\beta \approx 3.4$. Hence

$$P_{\sqrt{N}}\{y \equiv \sqrt{N} > x\} \sim x^{-2\beta} \tag{10}$$

with $2\beta \approx 6.8$. Therefore, N alone cannot explain the value $\alpha \approx 3$. Instead, $\alpha \approx 3$ must arise from elsewhere. In fact, when we repeat the analysis through to $W_{\Delta t}$ [19], we find that the distribution

$$P_W\{W_{\Delta t} > x\} \sim x^{-\gamma} \tag{11}$$

decays with an exponent $\gamma \approx 3$, which is also the contribution of W to the distribution $P_{W\sqrt{N}}$. Therefore, the averaging in Eq. (9) gives the asymptotic behavior of P_G to be a power-law with an exponent γ. Indeed, our mean estimates of γ and α are comparable within error bounds [12,19]. Thus, the power-law tails of $P_G(G)$ appear to originate from the power-law tail in $P_W(W)$.

We also analyze correlations in N. Instead of analyzing the correlation function directly, we use the method of detrended fluctuation analysis [20]. We plot the detrended fluctuation function $F(\tau)$ as a function of the time scale τ. Absence of long-range correlations would imply $F(\tau) \sim \tau^{0.5}$, whereas

$$F(\tau) \sim \tau^{\nu} \tag{12}$$

with $0.5 < \nu \leqslant 1$; this implies a power-law decay of the correlation function,

$$\langle [N(t)][N(t+\tau)] \rangle \sim \tau^{-\nu_{cf}}; \quad [\nu_{cf} = 2 - 2\nu]\,. \tag{13}$$

We obtain the value $\nu \approx 0.85$ for the same five stocks as before. On extending this analysis for a set of 1000 stocks, we find the mean value $\nu_{cf} \approx 0.3$ [19]. It is possible to relate this to the correlations in $|G|$, which is related to the variance V^2 of G. From Eq. (4), we see that $V^2 \propto N$ under the assumption that δp_i are independent. Therefore, the long-range correlations in N is one reason for the observed long-range correlations in $|G|$. In other words, highly volatile periods in the market persist due to the persistence of trading activity.

Naturally, the mechanisms that give rise to the observed long-range correlations in N are of great interest. In Ref. [21], this problem is investigated using a continuous time asynchronous model. Recently, it was argued that these correlations could arise from the fact that agents in the market have the choice between active and inactive strategies [22].

Finally, we discuss the role of the share volume traded to explain the statistical properties of price fluctuations. Intuitively, one expects that the larger the trade size, the greater the price impact, and hence the larger the volatility. Therefore, one expects the volatility to be related to the number of shares traded (share volume). Indeed, it is a common Wall Street saying that "it takes volume to move stock prices". We find [23] that the number of shares q_i traded per trade has a power-law distribution with tail-exponents ζ which are in the Lévy stable domain. Therefore, one can express the number of shares Q traded in Δt as

$$Q = \sum_{i=1}^{N} q_i \, . \tag{14}$$

Due to the Lévy stable tails of the distribution of q, Q scales like

$$Q = \mu N + N^{1/\zeta} \xi \, , \tag{15}$$

where ξ is a one-sided Lévy stable distributed variable with zero mean and tail exponent ζ, and $\mu \equiv \langle q_i \rangle$.

Analyzing equal-time correlations, we find, surprisingly, that the correlation coefficients $\langle \xi N \rangle$, $\langle \xi W \rangle$ are small (average values of the order of ≈ 0.1). This means that even if the number of shares traded are large (large ξ), volatility

$$V = W\sqrt{N} \tag{16}$$

need not be. Thus, the previously found [24–27] equal-time dependence of volatility $V = W\sqrt{N}$ and share volume arises largely because of N. This is quite surprising since it means that the size of the trade, on average, does not seem to have a direct influence in generating volatility [28].

3. Collective behavior of stock price movements

The problem of quantifying cross-correlations between the price movements of different stocks is important not only from the point of view of understanding collective behavior between the constituents of a complex system, but also from the point of view of estimating the risk of a investment portfolio. The usual way of quantifying cross-correlations is either by estimating the relevant "factors" or by principal component analysis [29]. Here, we review some results of a different approach to this problem by applying methods of random matrix theory [30–38].

In order to quantify correlations, we first calculate the price change ("return") of stock $i = 1, \ldots, N$ over a time scale Δt defined in Eq. (1). We analyze $L = 6448$ records 30-min price changes $G_i(t)$ for $N = 1000$ stocks (largest by market capitalization on 1 January 1994) for the two-year period 1994–1995. Since different stocks have varying levels of volatility (standard deviation), we define a normalized return

$$g_i(t) \equiv \frac{G_i(t) - \langle G_i \rangle}{\sigma_i} \, , \tag{17}$$

where $\sigma_i \equiv \sqrt{\langle G_i^2 \rangle - \langle G_i \rangle^2}$ is the standard deviation of G_i, and $\langle \cdots \rangle$ denotes a time average over the period studied. We then compute the equal-time cross-correlation matrix C with elements

$$C_{ij} \equiv \langle g_i(t) g_j(t) \rangle \, . \tag{18}$$

By construction, the elements C_{ij} are restricted to the domain $-1 \leqslant C_{ij} \leqslant 1$, where $C_{ij} = 1$ corresponds to perfect correlations, $C_{ij} = -1$ corresponds to perfect anti-correlations, and $C_{ij} = 0$ corresponds to uncorrelated pairs of stocks. In matrix notation, the correlation matrix can be expressed as

$$\mathsf{C} = \frac{1}{L} \mathsf{G}\mathsf{G}^{\mathsf{T}} \, , \tag{19}$$

where G is an $N \times L$ matrix with elements $\{g_{im} \equiv g_i(m\Delta t);\ i = 1, \ldots, N;\ m = 0, \ldots, L-1\}$, and G^{T} denotes the transpose of G.

We analyze the distribution $P(C_{ij})$ of the elements $\{C_{ij};\ i \neq j\}$ of the cross-correlation matrix C. We first examine $P(C_{ij})$ for 30-min returns from the TAQ database for the 2-yr periods 1994–1995 and 1996–1997. We find, firstly, that $P(C_{ij})$ is asymmetric and centered around a positive mean value ($\langle C_{ij} \rangle > 0$), implying that positively correlated behavior is more prevalent than negatively correlated (anti-correlated) behavior. Secondly, we find that $\langle C_{ij} \rangle$ depends on time, e.g., the period 1996–1997 shows a larger $\langle C_{ij} \rangle$ than the period 1994–1995. We contrast $P(C_{ij})$ with a control—a correlation matrix R with elements R_{ij} constructed from $N = 1000$ mutually uncorrelated time series, each of length $L = 6448$, generated using the empirically found distribution of stock returns [12,11]. We find that $P(R_{ij})$ is consistent with a Gaussian with zero mean, in contrast to $P(C_{ij})$. In addition, we see that the part of $P(C_{ij})$ for $C_{ij} < 0$ (which corresponds to anti-correlations) is within the Gaussian curve for the control, suggesting the possibility that the observed negative cross-correlations in C may be an effect of randomness.

Although by construction the elements of C are supposed to express the pairwise correlations that exist in the system, in practice, their meaning is not clear because of the time average involved in their calculation. Time averaging over a finite time series introduces measurement "noise" whereas the use of long time series amounts to averaging over possibly changing correlations. This raises the following problem: how can we extract from C, the cross-correlations that are significant?

The approach followed here is to compare the empirical cross-correlation matrix C against the "null hypothesis" of a random matrix of the same type ("symmetry"). Therefore, we consider a random correlation matrix

$$\mathsf{R} = \frac{1}{L} \mathsf{A}\mathsf{A}^{\mathsf{T}} \, , \tag{20}$$

where A is an $N \times L$ matrix containing N time series of L random elements with zero mean and unit variance, that are mutually uncorrelated. By construction, R belongs to the type of matrices often referred to as Wishart matrices in multivariate statistics [39].

The comparison between C and R is performed in the diagonal basis. Thus, we first compute the eigenvalues λ_k and eigenvectors u^k, where $k = 1, \ldots, N$ is arranged in order of increasing eigenvalues. Statistical properties of the eigenvalues of random matrices such as R are known [40–42] in the limit of very large dimensions. Particularly, in the limit $N \to \infty$, $L \to \infty$, such that $Q \equiv L/N$ is fixed, it was shown analytically [41] that the distribution $P_{rm}(\lambda)$ of eigenvalues λ of the random correlation matrix R is given by

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} , \tag{21}$$

for λ within the bounds $\lambda_- \leqslant \lambda_i \leqslant \lambda_+$, where λ_- and λ_+ are the minimum and maximum eigenvalues of R respectively, given by

$$\lambda_{\pm} = 1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}} . \tag{22}$$

We now compare the eigenvalue distribution of C and compare against $P_{rm}(\lambda)$. First, we observe that the "bulk" of the eigenvalues of C are consistent with $P_{rm}(\lambda)$ [30,31]. This suggests the randomness of the bulk which can be tested more rigorously by comparing against universal features of eigenvalue correlations of real symmetric random matrices. Specifically, our examination of the eigenvalue spacing distribution shows good agreement with the results for real symmetric (GOE-type) random matrices.

Secondly, we find deviations from RMT for the largest few eigenvalues [32,33]. These deviations are also evident when one examines the distribution of eigenvector components [30,31]. We find that $\rho(u)$ for a typical u^k from the bulk shows good agreement with the RMT result $\rho_{rm}(u)$. Similar analysis on the other eigenvectors belonging to eigenvalues within the bulk yields consistent results, in agreement with the results of the previous sections that the bulk agrees with random matrix predictions. Consider next the "deviating" eigenvalues λ_i, larger than the RMT upper bound, $\lambda_i > \lambda_+$. For deviating eigenvalues, the distribution of eigenvector components $\rho(u)$ deviates systematically from the RMT result $\rho_{rm}(u)$.

Finally, we examine the distribution $\rho(u^{1000})$ of the components of the eigenvector u^{1000} corresponding to the largest eigenvalue λ_{1000}. We find that $\rho(u^{1000})$ deviates significantly from a Gaussian. Specifically, we observe from $\rho(u^{1000})$ that all stocks contribute almost equally, and the distribution is rather narrow, suggesting that this eigenvector represents a collective mode in which all stocks participate. This notion can be quantified by comparing the price fluctuations of the portfolio defined by the u^{1000} against a standard measure of the fluctuations of the entire market—the fluctuations of the S&P 500 index. This comparison results in an equal-time correlation coefficient of 0.85 showing good agreement [38]. Thus, the eigenvector corresponding to the largest eigenvalue represents a collective mode in which all companies participate.

The magnitude of the largest eigenvalue itself seems to reflect the degree of collective behavior, as can be seen by examining the time evolution of the largest eigenvalue. We consider daily price fluctuations of 422 stocks for the years 1962–1996, and examine the time evolution of the largest eigenvalue λ_{422} compared against the time evolution of the S&P 500 index and the S&P 500 volatility. The large downward movement of the index in 1987 corresponds to the 1987 crash, when all stocks in the market almost simultaneously lost value; i.e., all stocks were moving more synchronously than usual.

We can also examine the remainder of the eigenvalues. Our analysis [40] shows that the eigenvectors corresponding to these eigenvalues have significant participants that corresponds to major industry groups. Thus, remaining deviating eigenvectors quantify collective behavior of stocks belonging to the same or related industries. We also find that one of the deviating eigenvectors contains mainly stocks of firms having business in Latin America. It is possible that this collective behavior is related to the large currency-devaluation in Mexico during the end of 1994 [38]. Similar results were obtained by using ultra-metric concepts by Refs. [43,44].

These deviating eigenvectors also have interesting dynamical features. For example, we find that the price fluctuations corresponding to the portfolios defined by the deviating eigenvectors are characterized by time correlations that decay significantly slower than that for a random eigenvector or for an individual stock [38]. This is reminiscent of the phenomenon of critical slowing down where collective modes of the system display very large relaxation times in the vicinity of a critical point [45,46].

4. Some similarities with critical point phenomena

Just above, we mentioned one analogy between stock price fluctuations and dynamic critical phenomena. Recent work suggests there may be additional analogies. For example, it appears stock prices respond to fluctuations in demand, in a fashion that is remarkably parallel to the way the magnetization of an interacting spin system responds to fluctuations in the magnetic field. Periods with large number of market participants buying the stock imply mainly positive changes in price, analogous to a magnetic field causing spins in a magnet to align. Recently, Plerou et al. [47] addressed the question of how stock prices respond to changes in demand. They quantified the relations between price change G over a time interval Δt and two different measures of demand fluctuations: (a) Φ, defined as the difference between the number of buyer-initiated and seller-initiated trades, and (b) Ω, defined as the difference in number of shares traded in buyer and seller initiated trades. They find that the conditional expectations $\langle G\rangle_{\Phi}$ and $\langle G\rangle_{\Omega}$ of price change for a given Φ or Ω are both concave. They find that large price fluctuations occur when demand is very small—a fact which is reminiscent of large fluctuations that occur at critical points in spin systems, where the divergent nature of the response function leads to large fluctuations. Their findings are reminiscent of phase transitions in spin systems, where the divergent behavior of the response function at the critical point (zero magnetic field) leads to large fluctuations [48,1]. Further, Plerou et al. [49] find a curious "symmetry breaking" for values of Σ above a certain threshold value Σ_c; here Σ is defined to be the local first moment of the probability distribution of demand Ω, the difference between the number of shares traded in buyer-initiated and seller-initiated trades. This feature is qualitatively identical to the behavior of the probability density of the magnetization for fixed values of the inverse temperature.

References

[1] H.E. Stanley, Scaling, universality, and renormalization: three pillars of modern critical phenomena, Rev. Mod. Phys. 71 (1999) S358–S366.
[2] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, Cambridge, 1999 (Japanese Translation: Masumi Nakajima, Economist-sha, Tokyo, 2000).
[3] H. Takayasu, M. Takayasu, Keizai, Jouhou, Seimei no Rinkai Yuragi, Diamond, Tokyo, 2000 (English translation: Critical Fluctuations in Economics, Information and Life, Springer, Berlin, 2001).
[4] J.P. Bouchaud, M. Potters, Theory of Financial Risk, Cambridge University Press, Cambridge, 2000.
[5] H. Levy, M. Levy, S. Solomon, Microscopic Simulation of Financial Markets, Academic Press, New York, 2000.
[6] B.M. Roehner, Hidden Collective Factors in Speculative Trading, Springer, Berlin, 2001.
[7] W. Paul, A. Baschnagel, Stochastic Processes from Physics to Finance, Springer, Berlin, 1999.
[8] J. Voit, The Statistical Mechanics of Financial Markets, Springer, Berlin, 2001.
[9] B. Gutenberg, C.F. Richter, Seismicity of the Earth and Associated Phenomenon, 2nd Edition, Princeton University Press, Princeton, 1954.
[10] D.L. Turcotte, Fractals and Chaos in Geology and Geophysics, Cambridge University Press, Cambridge, 1992.
[11] T. Lux, The stable Paretian hypothesis and the frequency of large returns: an examination of major German stocks, Appl. Financial Econom. 6 (1996) 463–475.
[12] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, M. Meyer, H.E. Stanley, Scaling of the distribution of price fluctuations of individual companies, Phys. Rev. E 60 (1999) 6519–6529.
[13] U.A. Muller, M.M. Dacorogna, O.V. Pictet, Heavy tails in high-frequency financial data, in: R.J. Adler, R.E. Feldman, M.S. Taqqu (Eds.), A Practical Guide to Heavy Tails, Birkhäuser Publishers, Basel, 1998, pp. 83–311.
[14] C.W.J. Granger, Z. Ding, Varieties of Long Memory Models, J. Econometrics 73 (1996) 61.
[15] Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C.-K. Peng, H.E. Stanley, The statistical properties of the volatility of price fluctuations, Phys. Rev. E 60 (1999) 1390–1400.
[16] M. Lundin, M.M. Dacorogna, U.A. Muller, Financial markets tick by tick, Wiley, New York, 1999, p. 91.
[17] M. Levy, H. Levy, S. Solomon, Microscopic Simulation of Financial Markets, Academic Press, Boston, 2000.
[18] J.-P. Bouchaud, Power laws in economics and finance: some ideas from physics, Quant. Finance 1 (2001) 105–112.
[19] V. Plerou, P. Gopikrishnan, L.A.N. Amaral, X. Gabaix, H.E. Stanley, Diffusion and economic fluctuations, Phys. Rev. E 62 (2000) R3023–R3026.
[20] K. Hu, Z. Chen, P.Ch. Ivanov, P. Carpena, H.E. Stanley, Effect of trends on detrended fluctuation analysis, Phys. Rev. E 64 (2001) 011114-1–011114-19.
[21] M. Shatner, L. Muchnik, M. Leshno, S. Solomon, cond-mat/0005430.
[22] J.-P. Bouchaud, I. Giardina, M. Mezard, cond-mat/0012156.
[23] P. Gopikrishnan, V. Plerou, X. Gabaix, H.E. Stanley, Statistical properties of share volume traded in financial markets, Phys. Rev. E 62 (2000) 4493–4496.
[24] J. Karpoff, Price variability and volume: a review, J. Financial Quant. Anal. 22 (1987) 109.
[25] G. Tauchen, M. Pitts, The price variability–volume relationship on speculative markets, Econometrica 57 (1983) 485–505.
[26] T.W. Epps, M.L. Epps, The stochastic dependence of security price changes and transaction volumes: implications of the mixture-of-distributions hypothesis, Econometrica 44 (1976) 305–321.
[27] P.K. Clark, A subordinated stochastic process model with finite variance for speculative prices, Econometrica 41 (1973) 135–155.
[28] C. Jones, K. Gautam, M. Lipson, Transactions, volumes and volatility, Rev. Financial Stud. 7 (1994) 631–651.
[29] J. Campbell, A.W. Lo, A. MacKinlay, The Econometrics of Financial Markets, Princeton University Press, Princeton, NJ, 1997.
[30] L. Laloux, P. Cizeau, J.-P. Bouchaud, M. Potters, Noise dressing of financial correlation matrices, Phys. Rev. Lett. 83 (1999) 1467.

[31] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, H.E. Stanley, Universal and nonuniversal properties of financial cross-correlation matrices, Phys. Rev. Lett. 83 (1999) 1471.
[32] P. Gopikrishnan, B. Rosenow, V. Plerou, H.E. Stanley, Quantifying and interpreting collective behavior in financial markets, Phys. Rev. E 64 (2001) 035106-1–035106-4.
[33] B. Rosenow, V. Plerou, P. Gopikrishnan, H.E. Stanley, Random matrix theory and portfolio optimization, preprint.
[34] V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral, T. Guhr, H.E. Stanley, A random matrix approach to financial cross-correlations, Phys. Rev. E, submitted.
[35] S. Drozdz, F. Gruemmer, F. Ruf, J. Speth, Physica A 287 (2000) 440.
[36] J.D. Noh, Phys. Rev. E 61 (2000) 5981.
[37] M. Marsili, e-print cond-mat/0003241.
[38] P. Gopikrishnan, B. Rosenow, V. Plerou, H.E. Stanley, Quantifying and interpreting collective behavior in financial markets, Phys. Rev. E 64 (2001) 035106-1–035106-4.
[39] R. Muirhead, Aspects of Multivariate Statistical Theory, Wiley, New York, 1982.
[40] F.J. Dyson, The distribution of singular values of certain random matrices, Rev. Mex. Fís. 20 (1971) 231.
[41] A.M. Sengupta, P.P. Mitra, Distributions of singular values for some random matrices, Phys. Rev. E 60 (1999) 3389.
[42] T. Guhr, A. Müller-Groeling, H.A. Weidenmüller, Random-Matrix theories in quantum physics: common concepts, Phys. Reports 299 (1998) 189–425.
[43] R.N. Mantegna, Eur. Phys. J. B 11 (1999) 193.
[44] G. Bonanno, F. Lillo, R.N. Mantegna, e-print cond-mat/0009350.
[45] P.C. Hohenberg, B.I. Halperin, Rev. Mod. Phys. 49 (1977) 435.
[46] S.-K. Ma, Modern Theory of Critial Phenomena, Benjamin, Reading, MA, 1976.
[47] V. Plerou, P. Gopikrishnan, X. Gabaix, H.E. Stanley, Quantifying stock price response to demand fluctuations, cond-mat 0106657, preprint.
[48] H.E. Stanley, Introduction to Phase Transitions and Critical Phenomena, Oxford University Press, Oxford, 1971.
[49] V. Plerou, P. Gopikrishnan, H.E. Stanley, Critical threshold and symmetry breaking in stock demand, preprint.