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Asymmetrical singularities in real-world signals

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We generalize the wavelet transform modulus maxima approach in order to analyze positive and negative changes separately and show different singularity spectra depending on the direction of changes in (i) human heartbeat interval data during sympathetic blockade, (ii) time series of daytime human physical activity of healthy individuals (but not of patients with debilitating fatigue), and (iii) daily stock price records of the Nikkei 225 in the period 1990–2002—but not of the S&P 500. We conclude that the analysis of asymmetrical singularities provides deeper insights into the underlying complexity of real-world signals that can greatly enhance our understanding of the mechanisms determining the systems' dynamics.

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Scale-invariant dynamics with power-law decaying correlations are ubiquitous in the physical sciences [1,2], and in a large number of real-world complex signals generated by physiologic [3–5] and economic [7] systems. Global and local scaling exponents have been used to quantify, respectively, the monofractal and multifractal singular behaviors of these scale-invariant dynamics [1,2]. These studies examined the data assuming that positive and negative changes have identical scaling properties. However, it is plausible that some real-world signals have *different* singular behaviors depending on the *direction* of changes. Here, we generalize the wavelet transform modulus maxima (WTMM) approach [8] in order to analyze scaling properties of positive and negative changes separately.

It has been demonstrated that a multifractal analysis using wavelets [9] is particularly adequate to the study of nonstationary signals [8] such as those encountered in physiological systems [4–6]. For example, the use of the nth derivative of the Gaussian function as a mother wavelet leads to the filtering of local $(n-1)$th order polynomial trends, enhancing the detection of singular behavior in time series [8]. In the WTMM method [8], a partition function $Z_q(a)$ is defined as the sum of the qth powers of the local maxima of the *modulus* of the wavelet transform coefficients at scale a. The power-law scaling of $Z_q(a)$ yields for small a the scaling exponents $\tau(q)$—the multifractal spectrum. The multifractal spectrum is related to the singularity spectrum $D(h)$, where $D(h_o)$ is the fractal dimension of the subset of the original time series characterized by a *local* Hurst exponent $h=h_o$ [1,2], through a Legendre transform [1], $D(h)=qh-\tau(q)$ with $h=d\tau(q)/dq$.

The WTMM method was successfully applied to the nonstationary healthy human heart interbeat intervals to demonstrate that heartbeat dynamics have multifractal properties [4]. Amaral *et al.* [5] further showed that pharmacological blockades of each of the sympathetic and parasympathetic branches of the autonomic nervous system controlling heartbeat intervals resulted in a significant loss of multifractality in human heartbeat dynamics. However, a fact not captured by these multifractal analyses is the difference between the "burstlike" activities of the sympathetic nerves [10]—whose occasional activation may result in a forceful decrease in heartbeat intervals—and the more steady action of the parasympathetic system which results in a more continuous increase or decrease of the heartbeat intervals. Motivated by the possibility of asymmetrical singularities in real-world signals, we define *two* partition functions, one obtained from the positive local maxima

$$Z_q^M(a)=\sum_{b\epsilon\{\max T\}} |\sup_{a'<a}\{T_\psi[f](a',b)\}|^q \qquad (1)$$

and the other obtained from the negative local minima

$$Z_q^m(a)=\sum_{b\epsilon\{\min T\}} |\inf_{a'<a}\{T_\psi[f](a',b)\}|^q, \qquad (2)$$

where $T_\psi[f](a,b)$ is the coefficient of the wavelet decomposition of the signal $f(t)$ at time $t=b$ for scale a [Figs. 1(a) and 1(b)].

First, we study the singular behavior of human heartbeat intervals during autonomic blockade. We analyze eight data sets obtained from six healthy subjects who were administered a placebo or a β-blocking drug, which reduces sympathetic control by blocking action of sympathetic neurotransmitters at the heart. Details of the subjects, protocols, and data collections are described by Amaral *et al.* [5]. As in previous studies [4,5], we used the third derivative of the Gaussian as the mother wavelet. Figure 1(a) displays part of a data set for one of the subjects during sympathetic blockade. The figure illustrates that the local minima of the wavelet transform coefficients detect periods of decreasing heartbeat intervals while the local maxima detect periods of increasing heartbeat intervals.

We calculate $Z_q^M(a)$ and $Z_q^m(a)$ for all data sets for sympathetic blockade and placebo administration [11]. As re-

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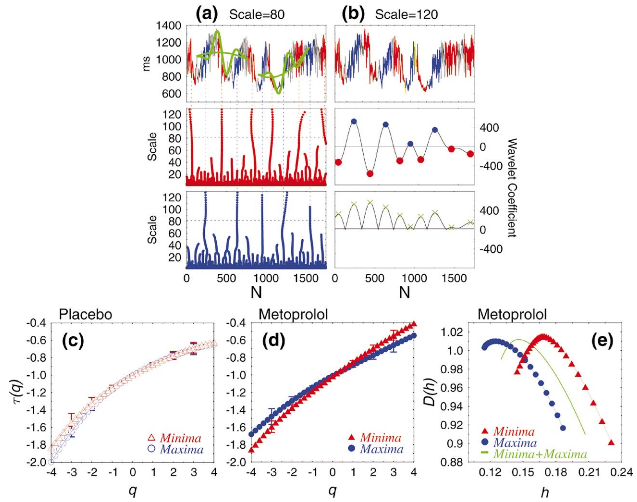

FIG. 1. (Color) (a) Heartbeat intervals in milliseconds vs beat number (top) for a subject administered a sympathetic blocker (metoprolol tartrate). The green curves represent the wavelet function, the third derivative of the Gaussian, for a scale $a=80$ and the local quadratic trend. The data are colored blue (red) for regions comprising 100 beats around local maxima (minima) detected for $a=80$. The data are colored yellow when maxima and minima regions overlap. In the middle and bottom panels, we plot the minima and maxima lines, respectively, of the coefficients of the wavelet transform. (b) Same data as in (a) but with maxima and minima detected for $a=120$ (top). The width of the colored regions is now 150 beats. The values of the coefficients of the wavelet transform of the time series are plotted in the middle panel with local maxima (minima) indicated in blue (red). The moduli of the coefficients of the wavelet transform of the time series are plotted in the bottom panel with local maxima indicated in green. (c, d) Multifractal spectra for healthy individuals administered (c) a placebo or (d) a sympathetic blocker. The vertical bars in the multifractal spectra represent the standard error of the mean for each condition. (e) Singularity spectra for the subjects under sympathetic blockade. The singularity spectra were calculated for values $-3<q<3$. For comparison, we also show in green the singularity spectra obtained using the WTMM method of Muzy *et al.* [8].

ported in Ref. [5], we find that the multifractal spectrum $\tau(q)$ during the sympathetic blockade [Fig. 1(d)] is significantly more linear than during placebo administration [Fig. 1(c)] [12]. This result implies a narrower singularity spectra $D(h)$ indicative of decreased multifractality. However, a fact not observed earlier is that the maxima and minima have different scaling properties during sympathetic blockade. Specifically [Fig. 1(e)], the peak of $D^m(h)$ during the sympathetic blockade is significantly shifted to larger h than the peak for $D^M(h)$—$p<0.05$ for the "drug × direction" interaction by three-way analysis of variance ANOVA and $p<0.05$ by a *post hoc* paired t test. No such difference between $D^M(h)$ and $D^m(h)$ is observed during placebo administration.

It is known that the burstlike nature of efferent sympathetic nerve dynamics in humans [10] results in abrupt decreases in heartbeat intervals. It is thus plausible that the asymmetry in the singular properties of interbeat intervals may have its origin in the suppression of the abrupt changes with smaller h generated by the sympathetic system. To test this hypothesis, we examine the effect of the parasympathetic blocker atropine [5], which blocks the action of a parasympathetic neurotransmitter, on the singular properties of interbeat intervals. We find that during the blockade of the parasympathetic system there is *no asymmetry* in the singularity spectra between maxima and minima. Since the parasympathetic system is thought to act in a more steady fashion than the sympathetic system, our results thus suggest that the abrupt changes generated by the sympathetic system are necessary to maintain the symmetry in the singular behavior of healthy heartbeat dynamics.

In order to further test the usefulness of this analysis method, we next study time series of human physical activity measured by trunk accelerometry. Human physical activity is a combined output of physiological and psychological processes that have complex regulating mechanisms. For this reason, decreased or altered patterns of physical activity have been used to identify and diagnose patients with psychiatric diseases [13] or with illnesses of unknown etiology such as chronic fatigue syndrome (CFS) [14,15]. "Gross" measures such as total amount of activity and cumulative distribution of activity counts have been used in most of these studies but, recently, time series analyses of physical activity [16] revealed scale-invariant dynamics with power-law decaying correlations in their microstructures.

Here, we analyze daytime physical activity time series from ten CFS patients and eight healthy age- and gender-matched control subjects for five days; details of the subjects, protocols, and data collections are described elsewhere [15]. In this case, we apply the Gaussian third derivative as the mother wavelet to the integrated physical activities [Fig. 2(b)]. For this wavelet function, the local positive maxima (negative minima) probe, respectively, periods with higher (lower) levels of physical activity than surrounding periods [Fig. 2(a)]. We calculate $Z_q^M(a)$ and $Z_q^m(a)$ and estimate the exponents $\tau(q)$ for time scales $0.71\leqslant\log(a)<1.54$, which corresponds to periods shorter than 35 min [17]. For CFS patients, $D^M(h)$ and $D^m(h)$ are very similar, i.e., the singular properties of the physical activity records are symmetrical [Fig. 2(c)]. In contrast, the slope, $\Delta\tau(q)/\Delta q\approx h$, of the multifractal spectra for $0\leqslant q\leqslant 3$ shows for the healthy controls a statistically significant asymmetry [Fig. 2(d)]; $p<0.05$ for the "disease × direction" interaction by repeated ANOVA and $p<0.01$ by a post hoc paired t test. This asymmetry is also visible in the larger width of $D^M(h)$ for small h [Fig. 2(d), inset].

An implication of these results is that in physical activity time series of healthy individuals the singularity behaviors are asymmetrical: The periods with higher levels of physical activity (which correspond to local maxima of the coefficients of the wavelet decomposition) have smaller h, i.e., peak in a more abrupt way than periods with lower levels of physical activity. That is, our results may be interpreted as suggesting that for healthy individuals, periods of higher physical activity came about more suddenly and are finished more quickly, while for CFS patients the periods of higher physical activity are quite symmetric with the periods of lower physical activity, possibly due to the exaggerated fa-

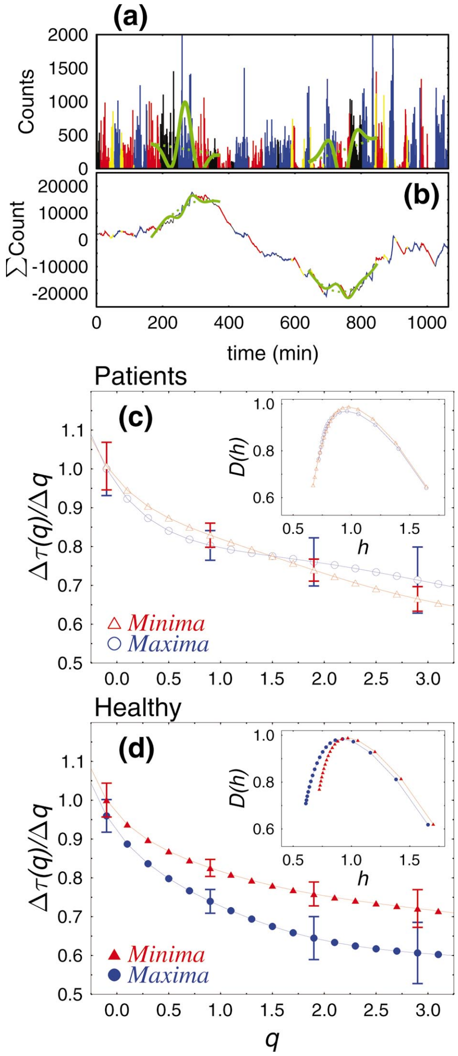

FIG. 2. (Color) (a, b) Daytime records of (a) physical activity and (b) integrated physical activity. The data are colored blue (red) in regions of width approximating 1 h centered at the locations of local maxima (minima) of the coefficients of the wavelet transform. The data are colored yellow when the regions for minima and maxima overlap. The green curves in (a) illustrate the wavelet function, the second derivative of the Gaussian, and the local linear trend. The green curves in (b) illustrate the wavelet function, the third derivative of the Gaussian, and the local quadratic trend. (c) Slope of the multifractal spectrum, $\Delta\tau(q)/\Delta q \approx h$, for $0 \leqslant q \leqslant 3$ for patients with chronic fatigue syndrome. The vertical bars represent the standard error of the mean. The inset displays the group mean singularity spectra obtained from $\tau(q)$ for $-3<q<3$. (d) Same as (c), but for healthy subjects.

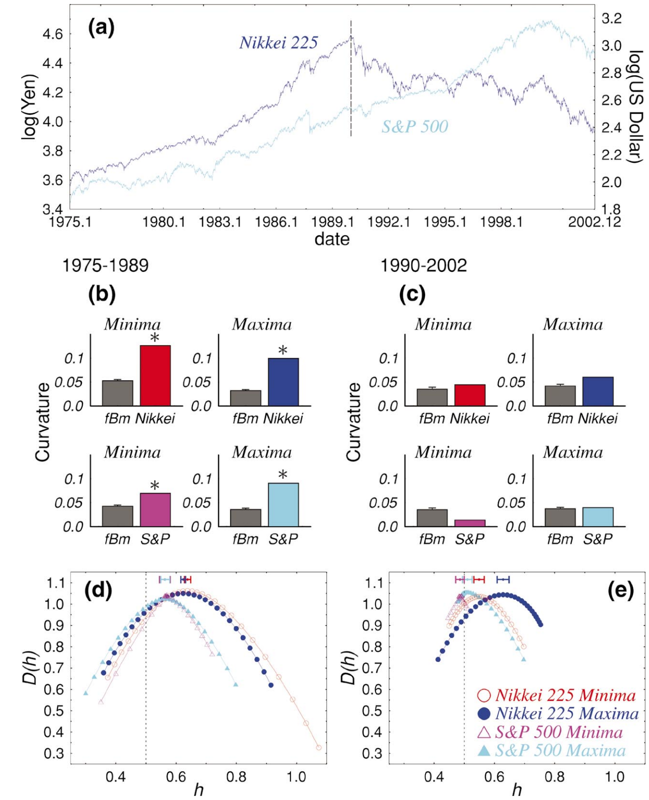

FIG. 3. (Color) (a) S&P 500 and Nikkei 225 daily closings for 1975–2002 period. (b,c) Curvature of $\tau(q)$ for the Nikkei 225 and the S&P 500 for the periods (b) 1975–1989 and (c) 1990–2002. (d,e) Singularity spectra for the Nikkei 225 and the S&P 500 for the periods (d) 1975–1989 and (e) 1990–2002. Horizontal bars in (d) and (e) represent the 95% confidence intervals of peak positions of $D(h)$ calculated from the monofractal fractional Brownian motion (fBm) surrogate data sets [20].

tigue they are afflicted with [18]. Our results thus suggest that by analyzing the singular behaviors of physical activity time series using local minima and maxima separately, one may obtain a new window into the study of disorders associated with debilitating fatigue.

As a further demonstration of the application of the analysis method proposed here, we study daily price records of U.S. (S&P 500) and Japanese (Nikkei 225) stock market indices for the period 1975–2002 [Fig. 3(a)]. We first examined the temporal evolution of peak locations of the singularity spectra $D^M(h)$ and $D^m(h)$ calculated for 10 year periods that are shifted forward in time by 3 year steps. To maximize the lengths of the time series for analysis and because the peak locations of $D(h)$ for the Nikkei 225 exhibit marked asymmetry (data not shown) after the crash of the Japanese stock market "bubble" in the early 1990s [dashed line in Fig. 3(a)], we split the data into two subsets: one for the period 1975–1989 and another for the period 1990–2002 [19].

For the period 1975–1989, we find that $\tau^M(q)$ and $\tau^m(q)$ for the Nikkei 225 and the S&P 500 are nonlinear [Fig. 3(b)], indicating multifractality [Fig. 3(d)] [20]. The peaks of $D(h)$ for the Nikkei 225 and S&P 500 are located at around $h=0.6$, larger than the value for Brownian noise, $h=0.5$ [21,22]. In contrast, for the period 1990–2002 we find that the multifractal spectra of the Nikkei 225 and S&P 500 data are more linear than those obtained for the period 1975–1989 [Fig. 3(c)]. For the S&P 500, the peaks of $D^M(h)$ and $D^m(h)$ are nearly identical and quite close to $h=0.5$ [Fig. 3(e)], indicating that there is neither correlation [21] nor asymmetry in the fluctuation of the S&P 500 for the studied period.

Most interestingly, we find that the peak of $D^M(h)$ for the Nikkei 225 after 1990 remains higher than $h=0.6$, while that of $D^m(h)$ is shifted toward $h=0.5$ [Fig. 3(e)]. These results suggest that after the crash, periods where the Nikkei 225 increases in value are associated with larger *h*, i.e., the index reaches local maxima very slowly, while periods when the Nikkei 225 decreases in value are associated with smaller *h*, indicating that the index quickly reaches a local low. These results provide insights into the dynamics during bear markets, where the gradual decrease in the actual stock price after the crash seems to be associated with the slower singularity behaviors only in the increasing direction.

In summary, as an extension of the wavelet-based analysis of multifractal singularity [8] only using characteristic points of interests (maxima and minima), we analyze *separately* singular behavior arising during increasing (maxima) and decreasing (minima) directions of change of a time series and are able to probe hitherto unobserved characteristics of complex time series generated by physiological and economic systems. We believe that the results made available by the technique proposed here provide greater opportunities to identify the mechanisms responsible for the observed dynamics and quantities with which to diagnose and prognose the state of complex systems.

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[11] We calculated $Z_q^M(a)$ and $Z_q^m(a)$ from data subsets, each with 30 000 point, for the sympathetic blockade and the placebo for scales $2<a<420$, and for moments $-4<q<4$. We then obtained the scaling exponents $\tau(q)$ from power-law fits to scales $1.31<\log(a)<2.56$. This range was selected because $\tau(0)$ is the closest to the theoretical value of -1.

[12] The curvature of $\tau(q)$, evaluated by the average $\Delta^2\tau(q)/\Delta q^2$ for $-3<q<3$, was significantly smaller for the sympathetic blockade than for the placebo ($p<0.001$ for the main effect of drug by three-way ANOVA).

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[17] The daytime records were collected at a 1 min sampling rate, so each record contains approximately 811 points. All quantities were calculated for each day and the results were then averaged for 5 days. This pooling of the data results in a total of approximately 4000 points per subject. We calculated first the average, the variance, and the global scaling exponent, calculated by the detrended fluctuation analysis [C.K. Peng *et al.*, Chaos **5**, 82 (1995)], of the activity signals and found no statistically significant difference between patients and controls.

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[19] The numbers of data points are $n=3791$ and $n=3278$ for S&P 500, and $n=4213$ and $n=3\,200$ for Nikkei 225, before and after January 1, 1990, respectively. We used the third derivative of the Gaussian to calculate the partition functions for scales $a=2-42$ and then estimated the exponents $\tau(q)$ within the average range of $0.47<\log(a)<1.54$.

[20] We compared the measured curvature of $\tau(q)$ for $-3<q<3$ for two stock price indices with those obtained from twenty subsets of four fractional Brownian motions (fBm) [21] with the same lengths and the same *h* values at the peak $D(h)$ as the data. We found that the curvatures of $\tau^M(q)$ and $\tau^m(q)$ for the Nikkei 225 and the S&P 500 data were significantly (*;$p<0.05$) greater than those for the monofractal surrogates for the period 1975–1989.

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