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Complex systems and networks: challenges and opportunities for chemical and biological engineers

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Abstract

The difference between the complicated and the complex is not just quantitative; it is also qualitative. Complexity requires both an augmentation of the conceptual framework and new tools. In this manuscript we describe the challenges faced when studying complex systems and describe how scientists from many different areas have responded to these challenges. We briefly describe the toolkit used for studying complex systems: nonlinear dynamics, statistical physics, and network theory. We place particular emphasis on network theory due to the explosive rate of advance that the field has recently experienced. We argue that chemical engineering—conversant with a systems viewpoint that is deeply embedded in its culture and the ability to tackle problems across a broad range of length and time scales—is in excellent position to master and develop new tools and to tackle the many challenges posed by complex systems. To illustrate this fact, we briefly review two cases—ecologic food webs and cellular networks—where chemical engineers could have an immediate impact.

Keywords: Complex systems; Networks; Nonlinear dynamics; Complexity; Small-world; Scaling; Universality

1. What is a complex system?

It is likely that if one brought together 10 researchers working on complex systems, one would end up with at least 11 definitions of what a complex system is. Researchers studying complex systems include physicists, ecologists, economists, engineers of all kinds, entomologists, computer scientists, linguists, sociologists, and political scientists. Considering this diversity, the cynics among us would likely conclude that the study of complex systems is an ill-defined area of study, while the enthusiast would likely counter that complex systems are such a broad area of research that it is difficult for the practitioners to converge on a single concise definition. Before trying to put forward a concise definition of what a complex system is, it might be worthwhile to distinguish between what we mean by *simple*, *complicated* and *complex*.

Simple systems have a small number of components which act according to well understood laws: Consider what is perhaps the prototypical simple system; the pendulum. The number of parts is small, in fact, one. The system can be described in terms of well-known laws—Newton's equations. The example of the pendulum raises an important point: The need to distinguish between *complex systems* and *complex dynamics*: It takes little for a simple system such as the pendulum to generate complex dynamics. A forced pendulum—with gravity being a periodic function of time—is chaotic. In fact one can argue that the driven pendulum contains everything that one needs to know about chaos; the entire dynamical systems textbook by Baker and Gollub (1990) is built around this theme. And a pendulum hanging from another pendulum—a double pendulum—is also chaotic (Fig. 1a).

Complicated systems have a large number of components which have well-defined roles and are governed by well-understood rules: A Boeing 747–400 has, excluding fasteners, 3×10^6 parts (Fig. 1b). In complicated systems, such as the 747, parts have to work in unison to accomplish a function. One key defect (in one of the many critical parts) brings the entire system to a halt. This is why redundancy is built into the design when system failure is not an option. More importantly, complicated systems have a limited

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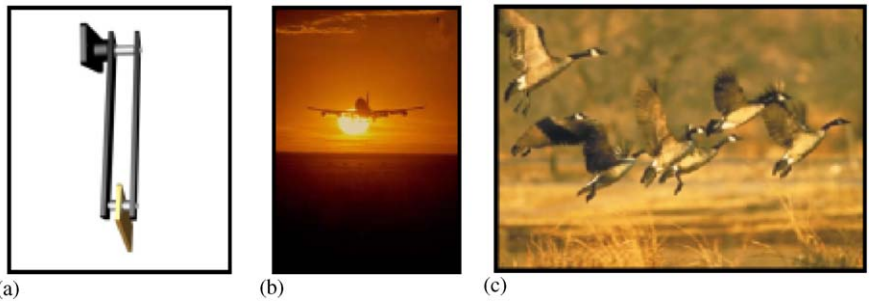

Fig. 1. Simple, complicated and complex systems. (a) The double pendulum—a pendulum hanging from another pendulum—is an example of a simple system. All parts can be well characterized and the equations describing their motion are also well known. (b) The Boeing 747–400 has on excess of 3×10^6 parts. (c) A flock of migrating geese. The Boeing is not a complex system because all its parts have strictly defined roles. This is typical of complicated systems, for which greater robustness is achieved through redundancy, i.e., including several copies of the same part in parallel. In contrast, for complex systems, robustness is achieved by enabling the parts to adapt and adopt different roles. The migrating geese provide a good example of such strategy, the ubiquitous "V" formations of the migrating geese are not static structures with a leader at the head, instead the structures are very fluid with a number of birds occupying the head position at different times.

range of responses to environmental changes. A 747 without its crew is not able to do much of anything to adjust to something extraordinary, and even the most advanced mechanical chronometers can only adjust to a small range of changes in temperature, pressure and humidity before they loose accuracy.

Complex systems typically have a large number of components which may act according to rules that may change over time and that may not be well understood; *the connectivity of the components may be quite plastic and roles may be fluid*: Contrast the Boeing 747–400 with a flock of migrating geese (Figs. 1b and c). Superficially, the geese are all similar and the flock has likely fewer members than the Boeing has parts, so one might be tempted to think that the Boeing is more complex than the flock of geese. However, the flock of migrating geese is an *adaptable* system, which the Boeing is not. The flock responds to changes in the environment—that is indeed why it migrates—moreover, and unlike what one may guess, the migrating geese self-organize without the need for a leader or maestro to tell the rest of the flock what to do. This is clearly revealed by observing the dynamic unrepeated patterns generated by the geese as they adjust their flying formations. Roles in the flock are fluid and one goose at the head of the formation will quickly be replaced by another. This feature of the flock gives it a great deal of robustness as no single goose is essential for the flock's success during the migration.

The stock market, a termite colony, cities, or the human brain, are also complex. As in the example of the flock of geese, the number of parts is not the critical issue. The key characteristic is adaptability. The systems respond to external conditions—a food source is obstructed and an ant colony finds a way to go around the object.

A working definition: *Self-organization and emergence*: Toulmin (1961) wrote about the creation of knowledge that: "Definitions are like belts. The shorter they are, the more elastic they need to be. [...] [A] short definition, applied to a heterogeneous set of examples, has to be expanded and contracted, qualified and reinterpreted, before it will fit every case. Yet the hope of hitting on some definition which is [...] satisfactory and brief dies hard..."

Agreeing on a concise definition may be difficult if not impossible. It is clear however that the hallmark of complex systems is the fact that (i) the units comprising the system are able to self-organize—as exemplified by the migrating geese—and (ii) out of the interaction of the units comprising the system something new is created—emergence. As the Nobelist Philip Anderson pointed out in his classical article "More is different" (Anderson, 1972), the interaction between a large number of units can give rise to totally different class of behaviors. Examples are among some of the most elusive and fascinating questions investigated by researchers nowadays: how consciousness arises out of the interactions of the neurons in the brain and between the brain and its environment, how humans create and learn societal rules, or how DNA orchestrates the processes in our cells.

Nonlinear interactions, one of the greatest challenges in the study of complex systems, are at the core of the emergence of qualitatively different states, new states that are not mere combinations of the states of the individual units comprising the system. The role of nonlinear dynamics on the understanding of complex systems has been common knowledge for more than two decades (Ottino, 2003).

Recently, a new aspect underlying the behavior of complex systems has been recognized—the structure of the network of interactions between the units comprising the system. As we will discuss in this paper, the realization of the importance of the network structure of complex systems may have provided the missing tool in the toolbox of complexity.

Objectives and organization of the manuscript: The goal of this manuscript is to argue that chemical engineering is in an excellent position to tackle the challenges posed by the study of complex systems and to master the use of the tools available for their study. In the remaining of the paper we describe the challenges faced when studying complex systems and describe how scientists from many different

areas have responded to these challenges. We also briefly describe the toolkit used for studying complex systems. Specifically we look at the three major categories of tools: nonlinear dynamics, statistical physics, and network theory. We place particular emphasis on network theory due to the explosive rate of advance that the field has experienced in the last 5 years. We will then briefly review two cases—ecologic food webs and cellular networks—where chemical engineers can have an immediate impact. Finally, we will discuss the need for a "new", or at very least, expanded definition of the meaning of prediction in the context of the study of complex systems.

2. Challenges in the study of complex systems

The units comprising a complex system do not have strictly defined roles, yielding a greater adaptability and robustness of the system. This feature of complex systems, however, increases the challenges in describing their structure and evolution. The prototypical challenges one faces when studying a complex system at various levels are:

The nature of the units: Complex systems typically comprise a large number of units, however, unlike the situation in many scientific problems, the units need not to be neither structureless nor identical.

Challenges:

- units have complex internal structures;
- units are not identical;
- units do not have strictly defined roles.

The nature of the interactions: Complex systems typically have units that interact strongly, often in a nonlinear fashion. Moreover, there are frequently stochastic components to the interaction and external noise acting on the system. An additional and crucial challenge is posed by the fact that the units are connected in a complex web of interactions that may be mostly unknown.

Challenges:

- nonlinear interactions;
- noise;
- complex network of interactions.

The nature of the forcing or energy input: Complex systems are typically out-of-equilibrium. For example, living organisms are in a constant struggle with their environment to remain in a particular out-of-equilibrium state, namely alive. Social and economic systems are also driven, out-of-equilibrium, systems; new technologies change the balance of power between companies, terrorist attacks change economic expectations, etc.

Challenges:

- poorly characterized distribution of external perturbations;
- poorly characterized correlations of external perturbations;
- nonstationarity of external perturbations.

3. Tools for the study of complex systems

In a rough sense, the current toolbox used in tackling complex systems involves three main categories (i) nonlinear dynamics, (ii) statistical physics, including discrete models, and (iii) network theory. Elements of nonlinear dynamics should be familiar to many of the readers of this journal (Doherty and Ottino, 1988). The one with perhaps the greatest degree of novelty to chemical engineers—because of the recent nature of most of the significant advances—is network theory, so we will try to provide a short introduction into the concepts and techniques of interest. First, however, we will quickly comment on the two other tool categories.

3.1. Nonlinear dynamics and chaos

Nonlinear dynamics and chaos in deterministic systems are now an integral part of science and engineering. The theoretical foundations are on firm mathematical footing. There are well agreed upon mathematical definitions of chaos, many of them formally equivalent. Because of its novelty and, in many case, counter-intuitive nature, there are still many misconceptions about chaos and its implications. Extreme sensitivity to initial conditions does not mean that prediction is impossible. Memory of initial conditions is lost within attractors but the attractor itself may be extremely robust. In particular chaotic does not mean unstable.

Chaos means that simple systems are capable of producing complex outputs. Simple 1D mappings can do this—the logistic equation being the most celebrated example. The flip side is that complex looking outputs need not have complex or even complicated origins; seemingly random-looking outputs can be due to deterministic causes. Many techniques have been developed to analyze signals and to determine if fluctuations stem from deterministic components.

Nonlinear dynamics is now firmly embedded throughout research; applications arise in virtually all branches of engineering and physics—from quantum physics to celestial mechanics. There are numerous applications in geophysics, physiology and neurophysiology (Glass and MacKey, 1988). Even sub-applications have developed into full-fledged areas. For example, mixing is one of the most successful areas of applications of nonlinear dynamics (Ottino, 1989). Within chemical engineering, successful applications have included mixing, dynamics of reactions, fluidized beds, pulsed combustors, bubble columns. For example, chemical reactions, in combination with diffusion, can be exploited to produce a dizzying array of structures (Kiss and Hudson, 2003).

It is clear that nonlinear dynamics does not exist in isolation but it is now a platform competency. This does not mean that all theoretical questions have been answered and

that all ideas are uncontroversial. For example there is significant discussion about the presence of chaos in physics and the role it may play in determining the universe's "arrow of time", the irreversible flow from the past to the future.

3.2. Statistical physics: universality and scaling

Of the three revolutionary new areas of physics born at the turn of the 20th century—statistical physics, relativity, and quantum mechanics, it is fair to say that statistical physics has been the area that least caught people's imagination. The reason may be that, on the surface, statistical physics most resembles pre-20th century physics. However, statistical physics brought three very important conceptual and technical advances:

(1) It lead to a new conception of prediction—cf. the Maxwell demon paradox. We shall have more to say about this change when we discuss the concept of prediction latter in the paper.
(2) It circumvented classical mechanics and the impossibility to solve the three-body problem by tackling the many-body problem. In doing so, it casted solutions in terms of ensembles.
(3) It introduced the concept of discrete models—ranging from the Ising model to cellular automata (Wolfram, 2002) and agent-based models (Epstein and Axtell, 1996).

In the 1960s and 1970s, fundamental advances occurred in our understanding of phase transitions and critical phenomena leading to the development of two important new concepts: universality and scaling (Stanley, 1971, 1999). The finding, in physical systems, of universal properties that are independent of the specific form of the interactions gives rise to the intriguing hypothesis that universal laws or results may also be present in complex social, economic and biological systems (see Fig. 2).

Indeed, recently it has come to be appreciated that many complex systems obey universal laws that are independent of the microscopic details. Findings in one system may translate into understanding of the behavior of many others. For example, fluctuations in physiologic outputs of healthy individuals display universal degree of correlations (Peng et al., 1995; Ivanov et al., 1999; Amaral et al., 2001a; Goldberger et al., 2002), as do fluctuations of financial assets (Pagan, 1996; Gopikrishnan et al., 1999; Plerou et al., 1999a; Muller et al., 1999). Similarly, it has been recently shown that scaling and universality hold for a broad range of human organizations (Stanley et al., 1996; Amaral et al., 1997; Lee et al., 1998; Plerou et al., 1999a, b; Amaral et al., 2001b; Stanley et al., 2002).

3.2.1. Scaling

The *scaling hypothesis* which arised in the context of the study of critical phenomena led to two categories of predictions, both of which have been remarkably well verified by a wealth of experimental data on diverse systems. The first category is a set of relations, called *scaling laws*, that serve to relate the various critical-point exponents characterizing the singular behavior of the order parameter and of response functions.

The second category is a *data collapse*, which can be explained in terms of the simple example of a liquid at the critical point. One writes the equation of state as a functional relationship of the form $\Delta\rho = \Delta\rho(p, \tau)$, where $\Delta\rho = \rho(\text{liquid}) - \rho(\text{gas})$, p is the pressure, and $\tau \equiv (T - T_c)/T_c$ is a dimensionless measure of the deviation of the temperature T from the critical temperature T_c. Since $\Delta\rho(p, \tau)$ is a function of two variables, it can be represented graphically as $\Delta\rho$ vs. τ for a sequence of different values of p. The scaling hypothesis predicts that all the curves of this family can be "collapsed" onto a single curve provided one plots not $\Delta\rho$ vs. τ but rather a *scaled* $\Delta\rho$ ($\Delta\rho$ divided by p to some power) vs. a *scaled* τ (τ divided by p to some different power).

The predictions of the scaling hypothesis are supported by a wide range of experimental work, and also by numerous calculations on model systems. Moreover, the general principles of scale invariance just described have proved useful in interpreting a number of other phenomena, ranging from elementary particle physics (Jackiw, 1972) and

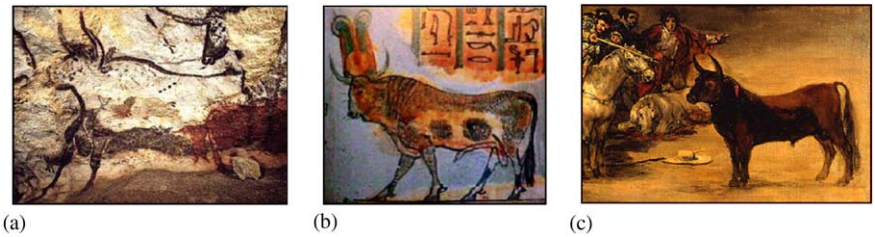

Fig. 2. Visualizing universality. (a) Lascaux cave paintings, beginning of Magdalenian Age (approximately 15,000–13,000 B.C.); (b) Apis bull, Egypt (3000–500 B.C.); (c) Bullfight: Suerte de vara (detail), Francisco de Goya y Lucientes (1824); oil on canvas (50 × 61 cm), The J. Paul Getty Museum, Los Angeles. Despite the difference in details, styles, and medium, all images are easily identified as despictions of bulls. Clearly, all images capture the essential characteristics of the animal. However, for a computer program, the task of classifying the subject matter of all pictures as being identical is far from trivial. The concept of universality in statistical physics and complex systems may aspires to the same goal as such a computer program would: to capture the essence of different systems and to classify them into distinct classes.

galaxy structure (Peebles, 1980) to finance and sociology (Amaral et al., 2001a; Stanley et al., 2002; Liljeros et al., 2003).

3.2.2. Universality

Another fundamental concept arising from the study of critical phenomena is *universality*. The idea has to do with taxonomy: One can classify all critical systems into "universality classes." Assume that one has the experimental $(\Delta\rho, p, T)$ data on five substances near their respective critical points. For each of the five substances, the data collapse onto a scaling function, supporting the scaling hypotheses. More remarkably, the scaling function is the *same* (apart from two substance-dependent scale factors) for all five substances.

The fact that the exponents and scaling functions are the same for all five substances implies they all belong to the same universality class. This fact suggests than when studying a given problem, one may pick the most tractable system to study and the results one obtains will hold for all other systems in the same universality class. This result has been demonstrated exactly for some physical systems and by renormalization group methods for others (Stanley, 1971, 1999).

This apparent universality of critical behavior motivated the search for the features of the microscopic interparticle force which are important for determining critical-point exponents and scaling functions, and which ones are unimportant. These questions were answered by numerous works on the renormalization group (Binney et al., 1992). The studies led to the idea that when the scale changes, the equations which describe the system also change accordingly and that in the macroscopic limit only a few "relevant" features remain. When one uncovers universality in a given system, it means that some profound, usually simple, mechanisms are at work. This conceptual framework has guided many physicists forays into interdisciplinary research yielding insights across seemingly dissimilar disciplines.

3.2.3. Discrete models

Discrete-space and discrete-time modeling is based on the assumption that some phenomena can and should be modeled directly in terms of computer programs (algorithms) rather than in terms of equations. Cellular automata—which can be traced to John von Neumann and Stanislaw Ulam and were further developed and popularized in Conway's "game of life" and, more recently, Wolfram—are the simplest example of discrete time and space models that were developed with the computer in mind.

Examples of the application of cellular automata exist in physical, chemical, biological and social sciences; they can be as simple as propagation of fire and simple predator–prey models between a handful of species and as complex as the evolution of artificial societies. The central idea is to have agents that live on the cells of regular d-dimensional lattices and interact with each other according to prescribed rules. The basic building blocks may be identical or may differ in important characteristics; moreover these characteristics may change over time, as the agents adapt to their environment and learn from their experiences—see e.g. Epstein and Axtell (1996) in the context of the social sciences.

Discrete, or agent-based, modeling has been extremely successful because of the intuition-building capabilities it provides and the speed with which it permites the investigation of multiple scenarios. For this reason discrete modeling has led in some cases to a replacement of equation-based approaches in disciplines such as ecology, traffic optimization, supply networks, and behavior-based economics. Applications of cellular automata to problems familiar to chemical engineers are somewhat more classical, involving fluid-flow and flow of granular matter (e.g. Peng and Herrmann, 1994, 1995; or Désérable, 2002).

4. Networks

It has recently become clear that neither random networks nor regular lattices are adequate frameworks within which to study the network of interactions among the units comprising "real-world" complex systems (Kochen, 1989; Watts, 1999; Newman, 2000; Strogatz, 2001; Albert and Barabási, 2002; Dorogovtsev and Mendes, 2002), including chemical-reaction networks (Alon et al., 1999; Jeong et al., 2000, 2001; Wagner and Fell, 2001; Ravasz et al., 2002; Milo et al., 2002; Oltvai and Barabási, 2002), neuronal networks (Koch and Laurent, 1999; Lago-Fernandez et al., 2000), food webs (Pimm et al., 1991; Paine, 1992; Camacho et al., 2002a,b; Dunne et al., 2002), social networks (Wasserman and Faust, 1994; Liljeros et al., 2001; Jin et al., 2001; Girvan and Newman, 2002; Watts et al., 2002; Newman, 2003), scientific-collaboration networks (van Raan, 1990; Newman, 2001), and the Internet and the World Wide Web (Faloutsos et al., 1999; Albert et al., 1999; Huberman and Adamic, 1999).

4.1. Network theory: a short history

The birth of network (or graph) theory links together two famous mathematicians: Euler and Erdös. The "conception" of the theory is universally attributed to Euler (1736) and his solution of the celebrated Königsberg bridge puzzle. As stated in Euler's manuscript: "In the town of Königsberg in Prussia there is an island A, called "Kneiphoff", with the two branches of the river (Pregel) flowing around it. There are seven bridges, a, b, c, d, e, f, and g, crossing the two branches. The question is whether a person can plan a walk in such a way that he will cross each of these bridges once but not more than once. [...] On the basis of the above I formulated the following very general problem for myself: Given any configuration of the river and the branches into which it may divide, as well as any number of bridges, to determine whether or not it is possible to cross each bridge exactly once."

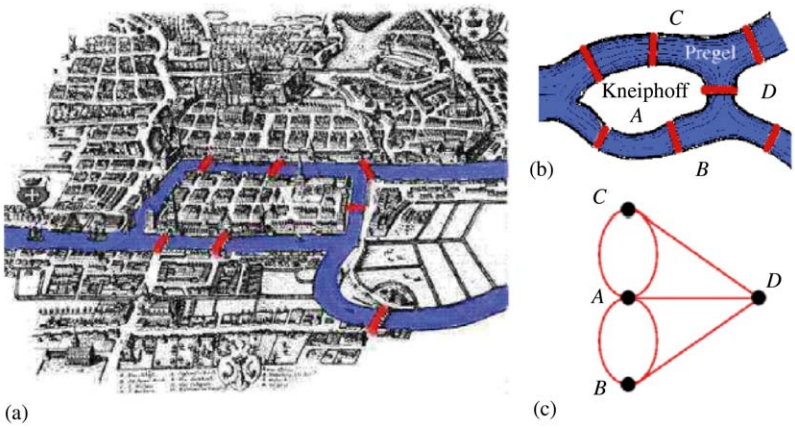

Fig. 3. The Königsberg bridge puzzle (Euler, 1736). (a) The town of Königsberg, now Kaliningrad, Russia, had at the time seven bridges connecting the island of Kneiphoff to the margins of the river Pregel. (b) Schematic representation of the area with the bridges. (c) Euler's representation of the problem. Euler realized that physical distance was of no importance in this problem, only topology matters. For this reason the bridges can be represented as links in a graph connecting nodes representing the different margins and islands.

Euler's solution of the Königsberg bridge puzzle developed naturally from his formulation of the problem, once again showing that formulation of a problem is as important, if not more than, the solution itself. Euler noticed that physical distance is of no importance in this problem and represented the topological constraints of the problem in the form of a graph—a set of nodes and the set of links connecting pairs of nodes (Fig. 3). Euler divided the nodes into odd and even based on the parity of the degree of the node, that is, the number of links directly connected to the node. He then demonstrated that

(1) the sum of degrees of the nodes of a graph is even;
(2) every graph must have an even number of odd nodes.

These results enabled him to show that

(1) if the number of odd nodes is greater than 2 no Euler walk exists—a Euler walk being a walk between two arbitrary nodes for which every link in the graph appears exactly once;
(2) if the number of odd nodes is 2, Euler walks exist starting at either of the odd nodes;
(3) with no odd nodes, Euler walks can start at an arbitrary node.

Therefore, since all four nodes in the Königsberg bridge problem are odd, Euler demonstrated that there was no solution to the puzzle, that is, there was no path transversing each bridge only once. Euler's work was of seminal importance because it identified topology as the key issue of the problem, thus enabling his later work on topology and the establishment of e.g. relations among the numbers of edges, vertices and faces of polyhedrons.

If the conception of network theory is due to Euler, its "delivery" is due in great part to Erdös. As in Euler's case, Erdös interest on network theory is linked to a social puzzle: What is the structure of social networks? This problem was formalized by Kochen and Pool in the a 1950s, leading them to the definition of random graphs (Kochen, 1989)—graphs in which the existence of a link between any pair of nodes has probability p. Erdös, in collaboration with Rényi, pursued the theoretical analysis of the properties of random graphs obtaining a number of important results, including the identification of the percolation threshold—that is, the average number of links per node necessary in order for a random graph to be fully connected—or the typical number of intermediate links in the shortest path between any two nodes in the graph.

4.2. Small-world networks

Kochen and Pool's work, which was widely circulated in preprint form before it finally was published in 1981 (Kochen, 1989), was a percursor to experimental work that lead to the discovery of the so-called six-degrees of separation phenomenon, later popularized in a homonym play by John Guare. The six-degree of separation phenomenon is typically referred to in the scientific literature as the small-world phenomenon (Milgram, 1967; Travers and Milgram, 1969).

A recurrent characteristic of networks in complex systems is the small-world phenomenon, which is defined by the co-existence of two apparently incompatible conditions, (i) the number of intermediaries between any pair of nodes in the network is quite small—typically referred to as the six-degrees of separation phenomenon—and (ii) the large local cliquishness or redundancy of the network—i.e., the large overlap of the circles of neighbors of two network neighbors. The latter property is typical of ordered lattices, while the former is typical of random graphs (Bollobás, 1985).

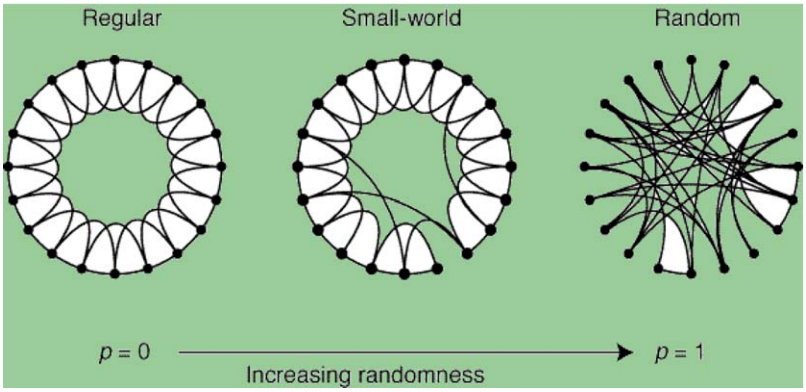

Fig. 4. A minimal model for generating small-world networks. Watts and Strogatz construct networks that exhibit the small-world phenomenon by randomizing a fraction p of the links connecting nodes in an ordered lattice. In the case displayed, the ordered lattice is one-dimensional with 4 connections per node. After Watts and Strogatz (1998).

Recently, Watts and Strogatz (1998) proposed a minimal model for the emergence of the small-world phenomenon in simple networks. In their model, small-world networks emerge as the result of randomly rewiring a fraction p of the links in a d-dimensional lattice (Fig. 4). The parameter p enables one to continuously interpolate between the two limiting cases of a regular lattice ($p=0$) and a random graph ($p=1$).

Watts and Strogatz probed the structure of their small-world network model and of real networks *via* two quantities: (i) the mean shortest distance L between all pairs of nodes in the network, and (ii) the mean clustering coefficient C of the nodes in the network. For a d-dimensional lattice one has $L \sim N^{1/d}$ and $C = O(1)$, where N is the number of nodes in the network. In contrast, for a random graph one has $L \sim \ln N$ and $C \sim 1/N$. Fig. 5a shows the dependence of L and C on p for the small-world model of Watts and Strogatz. The emergence of the small-world regime is clear for $p > 0.01$, as L quickly converges to the random graph value, while C remains in the ordered graph range, this two characteristic defining a small-world network. Watts and Strogatz (1998) found clear evidence of the small-world phenomenon for (a) the electric-power grid for Southern California, (b) the network of movie-actor collaborations, and (c) the neuronal network of the worm *C. elegans*.

A question is prompted by the results of Fig. 5a: "Under which conditions does the small-world regime emerge?" Specifically, does the small-world behavior emerge for a finite value of p when N approaches the thermodynamic limit? (Barthélemy and Amaral, 1999). Numerical results and theoretical arguments show that the emergence of the small-world regime occurs for a value of p that approaches zero as N diverges (Barthélemy and Amaral, 1999; Barrat and Weigt, 2000); cf. Fig. 5b. The implications of this finding are quite important: Consider a system for which there is a finite probability p of random connections. It then follows that independently of the value of p, the network will be in the small-world regime for systems with size $N \sim 1/p$, the reason being that to have a finite number of random links, i.e., that Np must be of $O(1)$. This implies that most large networks are small-worlds! Importantly, the nodes will be "un-aware" of this fact as the vast majority of them has no long-range connections (Barthélemy and Amaral, 1999).

4.3. Scale-free networks

An important characteristic of a graph that is not taken into consideration in the small-world model of Watts and Strogatz is the degree distribution, i.e., the distribution of number of connections of the nodes in the network. The Erdös–Rényi class of random graphs has a Poisson degree distribution (Bollobás, 1985), while lattice-like networks have even more strongly peaked distributions—a perfectly ordered lattice has a delta-Dirac degree distribution. Similarly, the small-world networks generated by the Watts and Strogatz model also have peaked, single-scale, degree distributions, i.e., one can clearly identify a typical degree of the nodes comprising the network.

Against this theoretical background, Barabási and co-workers found that a number of real-world networks have a scale-free degree distribution with tails that decay as a power law (Albert et al., 1999; Barabási and Albert, 1999). As shown in Figs. 6a–c, the network of movie-actor collaborations, the webpages in the `nd.edu` domain, and the power grid of Southern California, all appear to obey distributions that decay in the tail as a power law (Barabási and Albert, 1999). Moreover, other networks such as the network of citations of scientific papers also are reported to be scale-free (Seglen, 1992; Redner, 1998).

Barabási and Albert (1999) suggested that scale-free networks emerge in the context of growing network in which new nodes connect preferentially to the most connected nodes already in the network. Note that scale-free networks are a subset of all small-world networks because (i) the mean distance between the nodes in the network increases extremely slowly with the size of the network Barabási and Albert, 1999; Cohen and Havlin, 2003), and (ii) the

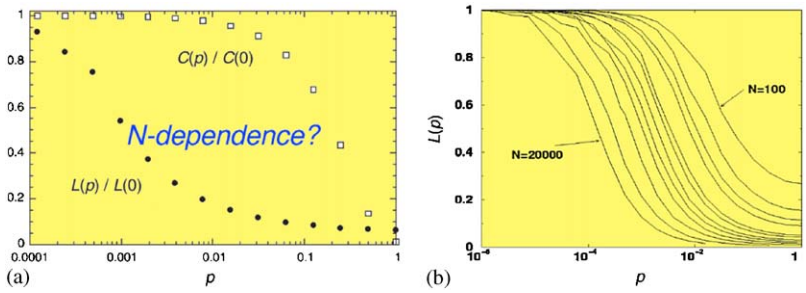

Fig. 5. Ubiquity of small-world networks. (a) Dependence of L and C on p for the small-world model of Watts and Strogatz. The emergence of the small-world regime is clear for $p > 0.01$, as $L(p)$ quickly converges to the random graph value, while $C(p)$ remains in the ordered graph range. After Watts and Strogatz (1998). (b) Dependence of L on p for different network sizes. The numerical results show that the emergence of the small-world regime occurs for a value of p that approaches zero as N diverges (Barthélemy and Amaral, 1999; Barrat and Weigt, 2000). After Barrat and Weigt (2000).

clustering coefficient is larger than for random networks. Importantly, scale-free networks provide extremely efficient communication and navigability as one can easily reach any other node in the network by sending information through the "hubs", the highly-connected nodes. The efficiency of the scale-free topology and the existence of a simple mechanism leading to the emergence of this topology led many researchers to believe in the absolute ubiquity of scale-free network. As it often happens, one finds what one is looking for!

4.4. Classes of small-world networks

An important aspect question prompted by the work of Barabási and Albert is how to connect the findings of Watts and Strogatz on small-world networks with the new finding of scale-free structures. Specifically, one may ask "Under what conditions will growing networks be scale-free?" or, more to the point, "Under what conditions will the action of the preferential attachment mechanism be hindered?" Recall that preferential attachment gives rise to a scale-free degree distribution in growing networks (Barabási and Albert, 1999), hence if preferential attachment is not the only factor determining the linking of incoming nodes one may observe other topologies. As is illustrated in Figs. 6d–f, Amaral and co-workers have demonstrated that preferential attachment can be hindered by at least three classes of factors:

Aging: This effect can be illustrated with the network of actors. In time, every actress or actor will stop acting. For the network, this implies that even a very highly connected node will eventually stop receiving new links. The node may still be part of the network and contributing to network statistics, but it no longer receives links. The aging of the nodes thus limits the preferential attachment preventing a scale-free distribution of degrees from emerging (Amaral et al., 2000).

Cost of adding links and limited capacity: This effect can be illustrated with the network of world airports. For reasons of efficiency, commercial airlines prefer to have a small number of hubs through which many routes connect. To first approximation, this is indeed what happens for individual airlines, but when we consider all airlines together, it becomes physically impossible for an airport to become a hub to all airlines. Due to space and time constraints, each airport will limit the number of landings/departures per hour, and the number of passengers in transit. Hence, physical costs of adding links and limited capacity of a node will limit the number of possible links attaching to a given node (Amaral et al., 2000).

Limits on information and access: This effect can be illustrated with the selection of outgoing links from a webpage in the World Wide Web: Even though there is no meaningful cost associated with including a hyperlink to a given webpage in one's own webpage, there may be constraints effectively blocking the inclusion of some webpages, no matter how popular and well connected they may been. An example of such constraints is distinct interest areas—a webpage on granular mixing is unlikely to include links to webpages discussing religion (Mossa et al., 2002).

As can be seen in Fig. 7, the presence of constraints leads to a cut-off of the power-law regime in the degree distribution, and that for a sufficiently strong constraints the power-law regime disappears altogether (Amaral et al., 2000). Empirical data suggest the existence of three classes of small-world networks (Amaral et al., 2000): (a) *scale-free* networks; (b) *broad-scale* or truncated scale-free networks, characterized by a degree distribution that has a power-law regime followed by a sharp cut-off that is not due to the finite size of the network; (c) *single-scale* networks, characterized by a degree distribution with a fast decaying tail, such as exponential or Gaussian. It is important to note that scale-free networks are small-world networks but the inverse may not be true!

5. Some possible chemical engineering applications

Mass and energy transport have been traditional domains of chemical engineering for five decades now. In many cases the topology of the system through which the transport is occurring is unimportant. In this section, we consider two

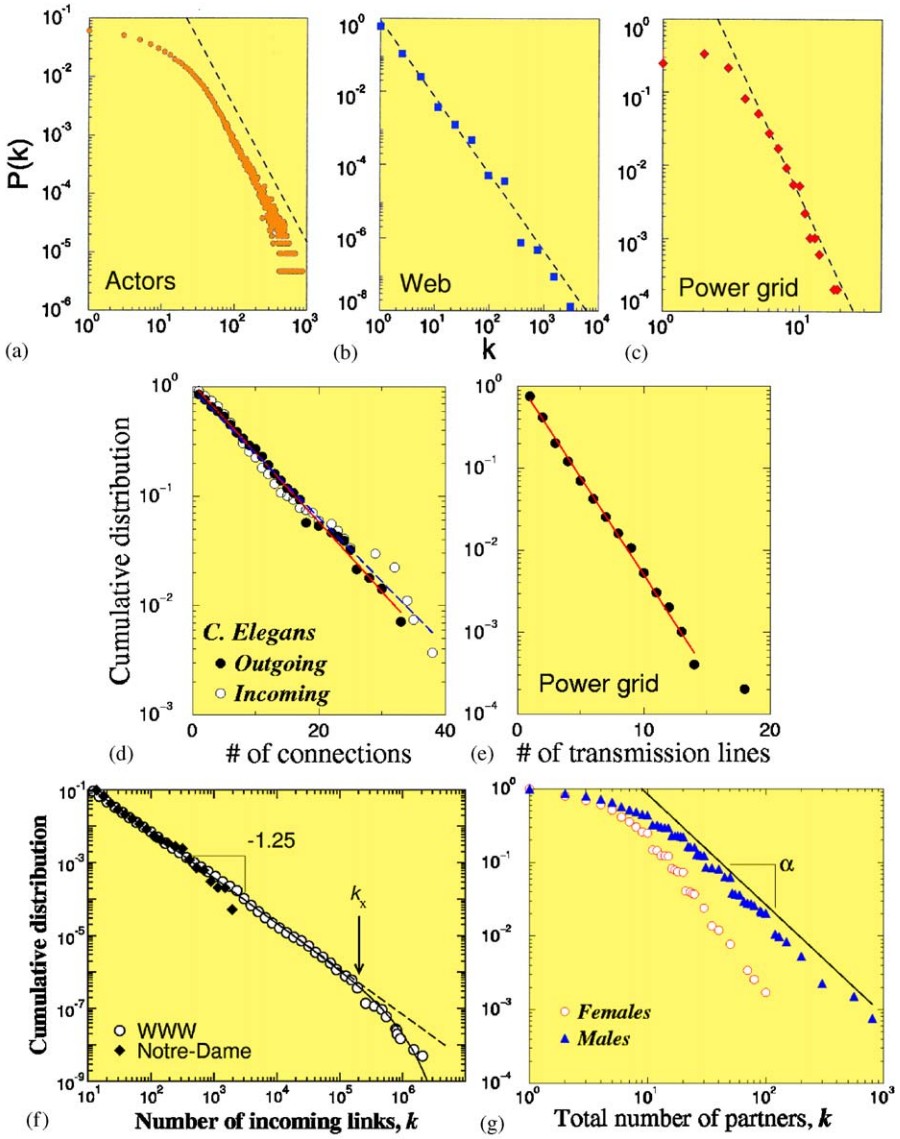

Fig. 6. Ubiquity of scale-free networks. Double logarithm plot of (a) the degree distribution of the network of movie-actor collaborations (each node corresponds to an actor and links between actors indicated that they collaborated on at least one movie); (b) the degree distribution of the webpages in the `nd.edu` domain (each node is a webpage and links between webpages indicate hyperlinks pointing to the other webpage); and (c) the degree distribution of the power grid of Southern California (each nodes is a transmission station and links are power lines connecting the stations). After Barabási and Albert (1999). (d) Degree distribution of the nematode *C. elegans*. Each of the 302 neurons of *C. elegans* and their connections has been mapped. Note that the plot is semi-logarithmic, so a straight line indicates an exponential dependence. After Amaral et al. (2000). (e) Degree distribution of the power grid of Southern California. Note that the data is much better described by an exponential decay then by a power law decay. After Amaral et al. (2000). (f) Degree distribution of the WWW. Note the truncation of the power law regime. After Mossa et al. (2002). (g) Distribution of number of sexual partners for Swedish females and males. Note the power law decay in the tails of the distributions. After Liljeros et al. (2001).

examples for which the way elements of the system are connected determines transport and the dynamics of the system.

5.1. The topology of natural ecosystems

Species in natural ecosystems are organized into complex webs. Ecologists have studied these webs from the perspective of network theory. Every species in the ecosystem being a node in a network and the existence of a trophic link—i.e., a prey–predator relationship—between two species indicating the existence of a *directed* link between them. We are far from this ideal, but understanding the structure of these food webs should be of fundamental importance in guiding policy decisions concerning, for example, the recommended limits on consumption of fish with high levels of pollutants, the selection of areas for establishment of protected ecosystems, or the management of boundary areas between protected ecosystems and agro-businesses.

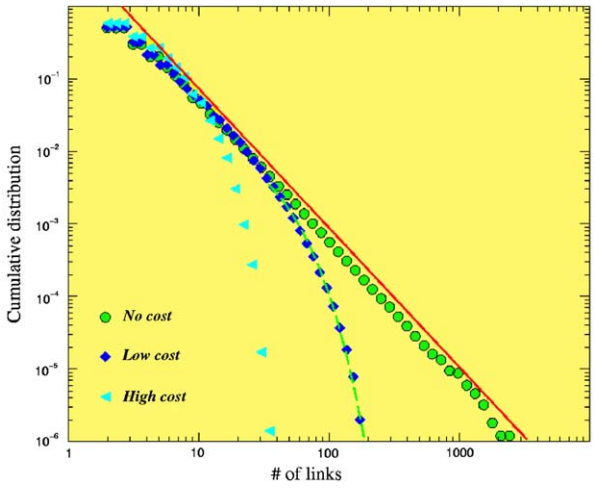

Fig. 7. Truncation of scale-free degree of the nodes by adding constraints to the model of Barabási and Albert (1999). Effect of cost of adding links on the degree distribution. These results indicate that the cost of adding links also leads to a cut-off of the power-law regime in the degree distribution, and that for a sufficiently large cost the power-law regime disappears altogether. After Amaral et al. (2000).

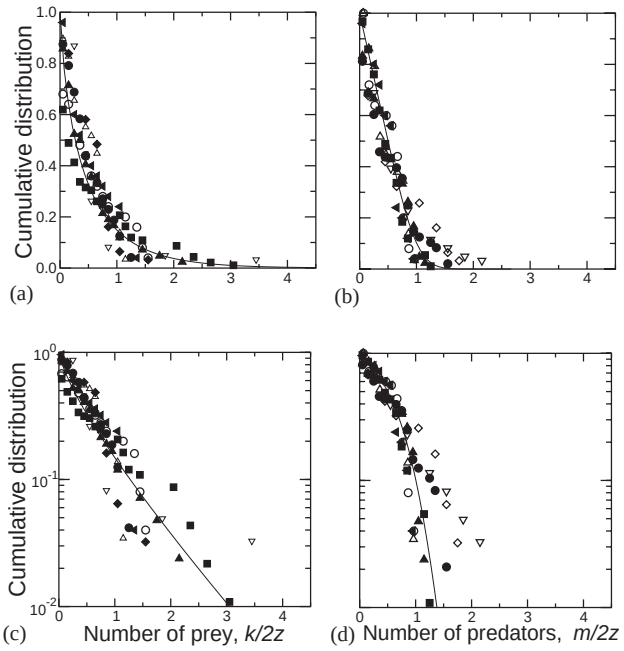

Fig. 8. Test of the "scaling hypothesis" that the distributions of number of prey (predators) have the same functional form for food webs from different environments. (a) Cumulative distribution P_{prey} of the scaled number of prey $k/2z$ for eight distinct food webs (see Camacho et al., 2004 for details). The solid line is the analytical prediction derived in Camacho et al. (2002a). The data "collapses" onto a single curve that agrees well with the analytical results derived in Camacho et al. (2002a). (b) Cumulative distribution P_{pred} of the scaled number of predators $m/2z$ for the same eight webs as in (a). The solid lines are the analytical predictions of. Semi-logarithmic plot of the scaled distributions of (c) number of prey, and (d) number of predators. After Camacho et al. (2004).

The study of such questions is extremely challenging for a number of reasons. First, the characterization of the topology of a given ecosystem is a very cumbersome and expensive task, which a priori may be of value only for the particular environment considered. Second, the precise modeling of the nonlinear interactions between the numerous individuals belonging to each of the many species comprising the ecosystem and the stochastic external variables (such as the climate) affecting the ecosystem may be impossible.

This topic has been outside of the province of chemical engineering. This, however, need not be the case. Consider for example, contaminant accumulation in aquatic species. Understanding of mass and energy balances, fluid dynamics and transport phenomena, statistical mechanics, kinetics, and applied mathematics are fundamental for the tackling of the problem on all relevant scales. When looked in its totality, the case for chemical engineers involvement is compelling.

Recently, Amaral and co-workers studied the *topology* of food webs from a number of distinct environments—including freshwater habitats, marine–freshwater interfaces, deserts, and tropical islands—and found that this topology may be identical across environments and described by simple analytical expressions (Camacho et al., 2002a,b, 2004). This finding is demonstrated in Fig. 8, where, as an example, we present the distributions of number of prey and number of predators for the species comprising eight distinct food webs.

In the same spirit, a recent paper in Nature reports on a study of food webs as transportation networks (Garlaschelli et al., 2003). The underlying idea is that the directionality of the links (pointing from prey to predator) defines a "flow" of resources—energy, nutrients, prey—between the nodes of the network. Because every species feeds directly or indirectly on environmental resources, food webs are connected (that is, every species can be reached by starting from an additional "source" node representing the environment. This fact enabled Garlaschelli et al. (2003) to define a spanning tree on any food web—i.e., a loopless subset of the links of the web such that, starting from the environment, every species can be reached. Importantly, they find that those spanning trees are characterized by universal scaling relations.

These results are of great practical and fundamental importance because they are consistent the hypothesis that scaling and universality hold for ecosystem—i.e., food webs display universal patterns in the way trophic relations are established despite *apparently* significantly differences in factors such as environment (e.g. marine versus terrestrial), ecosystem assembly, and past history. This fact suggests that a *general* treatment of the problems considered in environmental engineering may be within reach.

5.2. Cellular networks

The complexity of the web of nonlinear interactions between genes, proteins and the environment necessitates the

development of simplified models to illuminate biological function. As Vogelstein et al. (2000) wrote recently: "How can the vast number of activating signals, covalent and non-covalent modifications, and downstream regulators of p53 be put into context? One way to understand the p53 network is to compare it to the Internet. [...] An appreciation of the existence and complexity of cellular networks should enable more rational design and interpretation of experiments in the future, and should allow more realistic approaches to treatment."

A number of recent studies have indeed started to highlight *the existence and complexity* of cellular networks. Oltvai, Barabási and co-workers performed a systematic analysis of the metabolic networks of 43 organisms representing all three domains of life (Jeong et al., 2000). They found that, despite significant variation in their individual constituents and pathways, these metabolic networks have the same topological scaling properties and show striking similarities to the inherent organization of complex nonbiological systems. They concluded that metabolic organization is not only identical for all living organisms, but also complies with the design principles of robust and error-tolerant scale-free networks, and may represent a common blueprint for the large-scale organization of interactions among all cellular constituents (Jeong et al., 2000).

The same group also studied the protein–protein interaction network for two organisms, the yeast *S. cerevisiae* and the bacterium *H. pylori* (Jeong et al., 2001). They found that the network of protein interactions for these two organisms form a highly inhomogeneous scale-free network in which a few highly connected proteins play a central role in mediating interactions among numerous, less connected proteins.

Further, Jeong et al. (2001) tested the importance of the different proteins for the survival of the yeast by mutating its genome. For random mutations, they found that removal does not affect the overall topology of the network. However, they found that the likelihood that removal of a protein will prove lethal correlates with the number of interactions the protein has. For example, although proteins with five or fewer links constitute about 93% of the total number of proteins, they found that only about 21% of them are essential. By contrast, only some 0.7% of the yeast proteins with known phenotypic profiles have more than 15 links, but single deletion of 62% or so of these proves lethal. This implies that highly connected proteins are three times more likely to be essential than proteins with only a small number of links to other proteins.

In order to uncover the structural design principles of complex networks, Uri Alon and co-workers defined network motifs, patterns of interconnections occurring in real networks at numbers that are significantly higher than those in randomized networks (Milo et al., 2002). They found motifs in networks from biochemistry, neurobiology, ecology, and engineering. Remarkably, the motifs shared by ecological food webs were distinct from the motifs shared by the genetic networks of *E. coli* and *S. cerevisiae* or from those found in the World Wide Web. Similar motifs were found in networks that perform information processing, even though they describe elements as different as biomolecules within a cell and synaptic connections between neurons in the worm *C. elegans*.

Specifically, the two transcription networks and the neuronal connectivity network of *C. elegans* show the same motifs: a three-node motif termed "feedforward loop" and a four-node motif termed "bi-fan". The feedforward loop motif, in particular, may play a functional role in information processing. One possible function of this circuit is to activate output only if the input signal is persistent and to allow a rapid deactivation when the input goes off. Many of the input nodes in the neural feedforward loops are sensory neurons, which may require this type of information processing to reject transient input fluctuations that are inherent in a variable or noisy environment.

An area where chemical engineers are already contributing to a systems' approach to the study of cellular networks is the important work being done on metabolic engineering.

6. The meaning of prediction and the study of complex systems

Much discussion and debate, not always useful, has arisen when evaluating the fruits of a complex systems approach to problems. In our view, much of the disagreement is due to overly restrictive views of what is meant by prediction and what the limits to prediction are.

In order to put this question into perspective, let us examine the most usual meaning of prediction in the natural sciences, the "Newtonian" definition, put forward in its strongest form by Laplace and the meaning under which most scientists still operate today. In Newtonian physics one is able to predict the future and post-dict the past of any system for which one knows the position and velocity of all particles. A modern perspectic reveals a number of deficiencies.

First, it does not take into consideration computability issues. These are of two kinds. Assume one wants to compute the state of the entire universe, would not the "computer" be part of the system? Clearly, one cannot possibly model the behavior of the entire Universe, as that would not leave us with any material substrate with which to store the information or with which to perform the computation, for the same reason that one cannot draw a map that contains every detail of the real-world as the map must then be part of the map itself. Even if one would consider only a subset of the Universe, say, the water in a glass, one would still have to take into consideration the influence of the rest of the Universe on the water. One could easily model such influence as 'noise' acting on the system but that noise would destroy our ability to implement the Laplacian goal of predicting *exactly* the position and velocity of all particles.

Second, the study of deterministic nonlinear systems has clearly demonstrated the impossibility in exactly predicting of the velocities and positions of even simple systems interacting nonlinearly. The extreme dependence on initial conditions of chaotic nonlinear systems implies that in order to predict the positions and velocities of the units comprising a system interacting nonlinearly one would need to be able to measure initial velocities and positions exactly, a clearly unattainable goal even without considering quantum effects.

Moreover, even if Newtonian prediction was possible, it would not, in our view, convey in an enlightening and conceptual-building way the *relevant* information about the system. Consider again the water in a glass; it is clear that one can in principle determine the macroscopic state of the system—solid, liquid, or gas—and its temperature, volume and pressure from a complete description of the positions and velocity of the $O(10^{23})$ particles composing the system. However, would one *want* to do this? Clearly, the values of the macroscopic thermodynamic quantities provide a considerably more parsimonious description of the system. And, unarguably, the thermodynamic description of the system permits a deeper insight into the behavior of the system than the Newtonian approach of calculating forces and determining trajectories of all particles.

A more relevant class of what constitutes prediction in the context of the study of complex systems originated with developments in our understanding of phase transitions and critical phenomena. Close to the critical point most details of the system become irrelevant and the behavior of the systems is determined by a small number of relevant parameters and "mechanisms". For this reason, systems that may be very different in their details are actually described by the exact same scaling functions and sets of exponents (Stanley, 1971, 1999). A striking example of this type of prediction is the derivation of so-called allometric relationships: For example, the functional relationship between an organism's mass and its metabolic rate holds for organisms varying in mass over 27 orders of magnitude (; West et al., 1997; Banavar et al., 1999).

7. Concluding remarks

Dynamics and robustness of metabolic pathways, ecosystems, the web, and the US power grid; the propagation of HIV infections and the transfer of knowledge within organizations. These are all systems that fall within the scope of complex systems. The common characteristic of all complex systems is that they display organization without any external organizing principle being applied; a central characteristic is adaptability. The topic has already captured the attention of physics, biology and ecology, economics and social sciences. Where does engineering appear in this spectrum? And more specifically: Whats the role of chemical engineers?

In engineering, and chemical engineering in particular, we do both technology and science; we make and we explain. We explain (and model) phenomena and processes; we make materials and design processes. Some problems we pick; others are thrust upon us.

The hallmark of complex systems is adaptability and emergence: No one designed the web, the US power grid, or the metabolic processes within a cell. And this is where the conceptual conflict with engineering arises. Engineering is not about letting systems be (Ottino, 2004). The etymology of engineer, the verb and the noun, is revealing: ingenitor, contriver, ingenire, to contrive, as in to engineer a scheme. Engineering has a purpose and end result. Engineering is about convergence, assembling pieces that work in specific ways, optimum design and consistency of operation; the central metaphor is a clock. Complex systems, on the other hand, are about adaptation, self-organization and continuous improvement; the metaphor may be an ecology. It is robustness and failure where both camps merge. However, a successful merge will require augmenting the conceptual framework, even to the point of reshaping what one means by prediction.

In this paper we have focused on topological aspects of complexity; how agents are connected and what are the consequences of those interactions. An expansion of what constitutes complexity may be appropriate at this point. A complex system may alternatively be imagined as being comprised of a large number of units that interact with each other and with their environment; the interaction among units may be across length and time scales; the units can be all identical or different, they may move in space or occupy fixed positions, and can be in one state or multiple states.

Thus, if one follows this definition, then tools based on agent-based models come to the forefront. Alternatively one may define a complex system by (i) what it does: display organization without any organizing principle being applied, i.e. behavior emerges; or by (ii) how it may or may not be analyzed: decomposing the system and analyzing a part does not give a clue as to the behavior of the whole. This is probably the broadest definition: A complex system may be defined as a system that displays either (i) and/or (ii). Agent-based models and network theory contribute to explain (i); network theory provides tools on how to address some aspects (ii), as we have described in this article.

Many classical problems of engineering interest fall under (ii) as well. Granular dynamics provides an illustration. A classical example is the work of vibrated granular layers of (Umbanhowar et al., 1996) and the formation of oscillons. Detailed analysis of individual particles does not reveal the self-organization that takes place as the forcing of the system is changed. Other problems that squarely fit in this category are the many instances of multiscale modeling, cases where there is a linkage among a wide spectrum of length and time scales as in many cases of materials modeling, where there is a linkage between atomistic to macroscopic scales (Maroudas, 2000). Continuum physics provides other

examples that conceptually fit in the complex systems area, though it is unclear at the moment is any of the complex systems tools will be able to yield increased understanding of the main issues. Here we have in mind multiphase turbulent flow problems, where very strong fluctuations occur, and may be imagined as multiscale in both space and time, typically handled by means of coarse-graining.

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