

## Evolutionary game dynamics in a growing structured population

Julia Poncela<sup>1</sup>, Jesús Gómez-Gardeñes<sup>1,2</sup>, Arne Traulsen<sup>3,5</sup>  
and Yamir Moreno<sup>1,4</sup>

<sup>1</sup> Institute for Biocomputation and Physics of Complex Systems (BIFI),  
University of Zaragoza, E-50009 Zaragoza, Spain

<sup>2</sup> Departamento de Matemática Aplicada, ESCET, Universidad Rey Juan  
Carlos, E-28933 Móstoles (Madrid), Spain

<sup>3</sup> Emmy-Noether Group for Evolutionary Dynamics, Department of  
Evolutionary Ecology, Max Planck Institute for Evolutionary Biology,  
August-Thienemann-Strasse 2, 24306 Plön, Germany

<sup>4</sup> Department of Theoretical Physics, University of Zaragoza,  
Zaragoza 50009, Spain

E-mail: [traulsen@evolbio.mpg.de](mailto:traulsen@evolbio.mpg.de)

*New Journal of Physics* **11** (2009) 083031 (14pp)

Received 14 May 2009

Published 24 August 2009

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/11/8/083031

**Abstract.** We discuss a model for evolutionary game dynamics in a growing, network-structured population. In our model, new players can either make connections to random preexisting players or preferentially attach to those that have been successful in the past. The latter depends on the dynamics of strategies in the game, which we implement following the so-called Fermi rule such that the limits of weak and strong strategy selection can be explored. Our framework allows to address general evolutionary games. With only two parameters describing the preferential attachment and the intensity of selection, we describe a wide range of network structures and evolutionary scenarios. Our results show that even for moderate payoff preferential attachment, over represented hubs arise. Interestingly, we find that while the networks are growing, high levels of cooperation are attained, but the same network structure does not promote cooperation as a static network. Therefore, the mechanism of payoff preferential attachment is different to those usually invoked to explain the promotion of cooperation in static, already-grown networks.

<sup>5</sup> Author to whom any correspondence should be addressed.

**Contents**

<b>1. Introduction</b>	<b>2</b>
<b>2. Growing structured populations</b>	<b>2</b>
<b>3. Playing evolutionary games during growth</b>	<b>4</b>
<b>4. Promotion of cooperation in growing networks</b>	<b>6</b>
<b>5. Does cooperation benefit from growth or only from topology?</b>	<b>10</b>
<b>6. Discussion</b>	<b>10</b>
<b>Acknowledgments</b>	<b>12</b>
<b>References</b>	<b>12</b>

**1. Introduction**

Classical game theory is a branch of applied mathematics that has been developed to describe strategic interaction between fully rational individuals [1]. Evolutionary game theory is an elegant way to abandon the often problematic rationality assumption of classical game theory and to introduce a natural dynamics to that classical concept [2, 3]. In the past, evolutionary game theory has been used to describe either cultural learning dynamics or genetic reproduction under frequency dependent selection [4]. More recently, it has attracted a lot of interest in the physics community in the context of nonlinear dynamics [5, 6], disordered systems [7]–[9], finite size effects [10, 11], or spatially extended systems [12]–[18]. Statistical mechanics provides a powerful tool to describe evolutionary game dynamics in spatially extended, structured populations. Besides, in the last decade network theory has contributed significantly to our quantitative understanding of structured systems which go beyond the regularity of simple lattices [19].

A typical setup is the following: agents are assigned to the nodes of a network, which can be a regular lattice or have a more complex structure. Then, agents play an evolutionary game in which more successful strategies spread on the system. Describing these systems analytically is tedious and only possible in special cases [20]–[23]. Moreover, there are few general statements that can be made on evolutionary dynamics in such spatial systems [24].

Here, we drop another simplifying assumption and consider evolutionary games in growing, network-structured populations. In other words, instead of taking a growth algorithm for a particular network and later simulating evolutionary dynamics on that network, we grow the network while the evolutionary game is played. The interplay between growth and evolutionary game dynamics leads to interesting network structures and allows to disentangle effects based on topology from effects based on growth of the network.

**2. Growing structured populations**

We address the case of a growing population in which new individuals establish connections to the existing individuals, see also [25]. The newcomers can either connect to  $m$  arbitrary individuals or preferentially attach to those that have been successful players in the past. Success is based on the cumulated payoff  $\pi$  from an evolutionary game, which each individual plays with

all its neighbors on the network. For the model itself, we do not have to specify the kind of the game or the number of strategies.

We start from a small complete network of  $N_0$  individuals of one strategy. Subsequently, new individuals arrive and form connections to existing individuals. Evolutionary dynamics proceeds in the following ways:

- (1) In each time step, every individual  $j$  plays with all its neighbors and obtains an accumulated payoff  $\pi_j$ .

All players chose between their old strategy and the strategy of a randomly selected neighbor synchronously. Player  $j$  will adopt the strategy of its randomly selected neighbor  $i$  with probability

$$T_{j \rightarrow i} = \frac{e^{+\beta \cdot \pi_i}}{e^{+\beta \cdot \pi_i} + e^{+\beta \cdot \pi_j}}, \quad (1)$$

where  $\beta$  is the intensity of selection. With probability  $1 - T_{j \rightarrow i}$ , it will stick to its old strategy. For  $\beta \ll 1$ , selection is weak and the game is only a linear correction to random strategy choice. For strong selection,  $\beta \rightarrow \infty$ , it will always adopt a better strategy and it will never adopt a worse strategy. This process is routinely used in evolutionary game dynamics [13, 26, 27].

- (2) Every  $\tau$  time steps, a new individual with a random strategy is added to the system. For  $\tau \ll 1$ , several nodes are added before individuals change strategies. For  $\tau \gg 1$ , the network grows very slowly and the game dynamics can bring the system close to equilibrium before a new node is added. The new individual establishes  $m$  links to preexisting nodes, which are chosen preferentially according to their performance in the game in the last time step. Node  $j$  is chosen as an interaction partner with probability

$$p_j = \frac{e^{+\alpha \cdot \pi_j}}{\sum_{l=1}^N e^{+\alpha \cdot \pi_l}}, \quad (2)$$

where  $N$  is the number of nodes that already exist when the new node is added. The remaining  $m - 1$  links are added in the same way, excluding double links. For  $\alpha = 0$ , the newcomer attaches to a randomly chosen existing node. For small  $\alpha$ , attachment is approximately linear with payoff. For high  $\alpha$ , the newcomers will make connections to only very few nodes with high payoffs. For  $\alpha \rightarrow \infty$ , all newcomers will always attach to the  $m$  most successful players.

Since  $m$  links and a single node are added in each  $\tau$  time steps, the average degree of the network is given by

$$\frac{N_0(N_0 - 1)\frac{1}{2} + m\frac{t}{\tau}}{N_0 + \frac{t}{\tau}}, \quad (3)$$

where  $t$  is the number of time steps that has passed. Throughout this work, we will concentrate on  $m = 2$  and  $N_0 = 3$ .

Let us first focus on the simplest case in which each interaction leads to the same payoff, which we set to one. Then, the payoffs  $\pi_j$  are just the number of interactions an individual has, i.e. the degree  $\kappa_j$  of the node (normalizing by the degree of the node would essentially wash out the effect of the topology at this point [28, 29]). Evolutionary dynamics of strategies has no consequences and thus, the topology is independent of  $\beta$ . This allows us to discuss the growth

dynamics without any complications arising from the dynamics of strategies. We have several simple limiting cases:

- For  $\alpha = 0$ , the newcomer attaches at random to a new node. This leads to a network in which the probability that a node has  $k$  links decays exponentially fast with  $k$ . This situation corresponds to the case studied in [30], as individuals introduced earlier are likely to get more links. In this case, topology is independent of strategies for all intensities of selection  $\beta$  even when individuals play different strategies leading to different payoffs. Whenever  $\alpha > 0$ , there is an interplay between topological dynamics and strategy dynamics.
- For  $\alpha \ll 1$ , we can linearize  $p_j$ . In this case, we obtain

$$p_j = \frac{\alpha^{-1} + \kappa_j}{\sum_{k=1}^N (\alpha^{-1} + \kappa_k)}. \quad (4)$$

Thus, we recover the linear preferential attachment model introduced by Dorogovtsev *et al* [31]. When strategies differ in their payoffs, then not only the degree but also the strategy of the nodes and their neighbors will influence the probability to attach to a node.

- When  $\alpha$  is large, we will typically observe a network in which  $m$  of the  $N_0$  nodes of the initial complete network will be connected to almost all nodes that have been added during the growth stage. The emergence of these super-hubs hinges on the nonlinearity in equation (2).

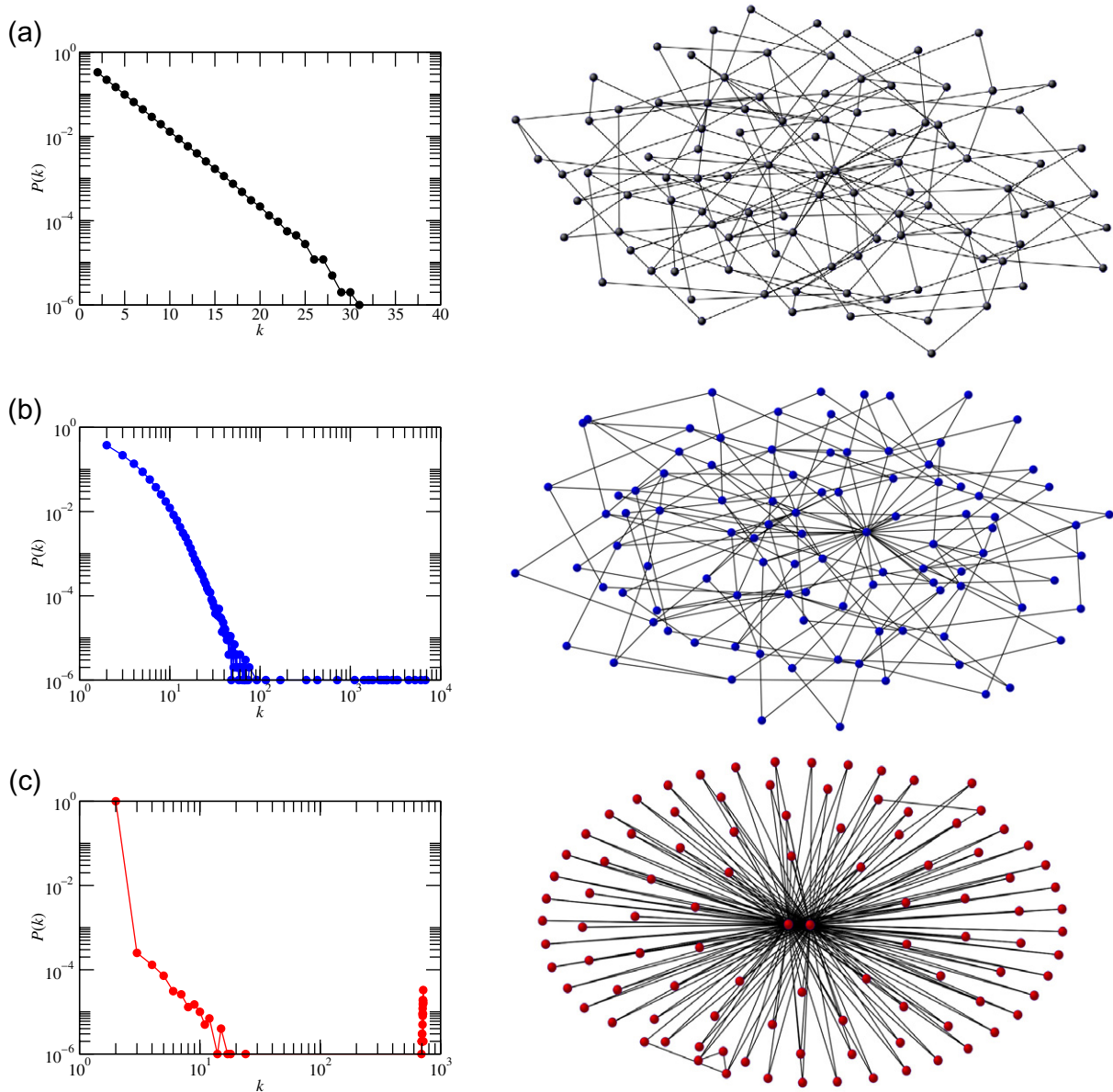
Examples for the network structures in these limiting cases are given in figure 1. Next, we turn to evolutionary games in which the payoff per interaction is no longer constant, but depends on the strategies of the two interacting individuals. In general, such an interplay of evolutionary dynamics of the strategies and the payoff-preferential attachment will change the structure of the network.

### 3. Playing evolutionary games during growth

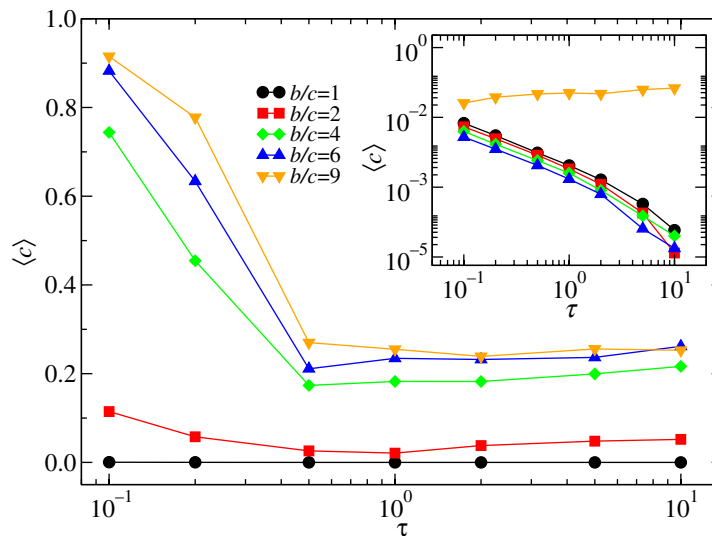
In principle, our framework allows to address any game between individuals, even repeated games or games with many strategies can be considered. However, we focus on the prisoner's dilemma here as an example of a one-shot game with two strategies [32]–[34]. Two players can choose between cooperation and defection. In the simplest case, there is a cost  $c$  to cooperation, whereas a cooperative act from an interaction partner leads to a benefit  $b$  ( $> c$ ). The game can be written in the form of a payoff matrix,

$$\begin{array}{cc} & \begin{array}{c} C \\ D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}. \end{array} \quad (5)$$

No matter what the opponent does, defection leads to a higher payoff (due to  $b > b - c$  and  $0 > -c$ ). Thus selfish, rational players should defect. Similarly, if the payoff determines reproductive fitness, evolution will lead to the spread of defection. However, the payoff for mutual defection is smaller than the payoff for mutual cooperation ( $b - c > 0$ ) and thus players face a dilemma. One way to resolve the dilemma is to consider structured populations in which players only interact with their neighbors [35]. Here, we follow this line of research and consider in addition growing populations, as discussed above.



**Figure 1.** Networks for a game in which both strategies have identical payoffs, such that the payoff is given by the degree of a node. The left-hand side shows the degree distributions of networks of size  $N = 10^4$ , while the right-hand side shows snapshots of networks of  $N = 100$  nodes. (a) For  $\alpha = 0.0$ , the degree distribution decays exponentially. (b) For  $\alpha = 0.1$ , some highly connected nodes appear in the network and the degree distribution begins to resemble a power-law. (c) Already for  $\alpha = 1.0$ , the vast majority of nodes ( $>99.9\%$ ) have only two links. In addition,  $m = 2$  of the  $N_0 = 3$  initial nodes are connected to almost all other nodes (degree distributions are obtained from an average over 100 networks, note that the  $x$ -axis is linear in (a), but logarithmic in (b) and (c)).



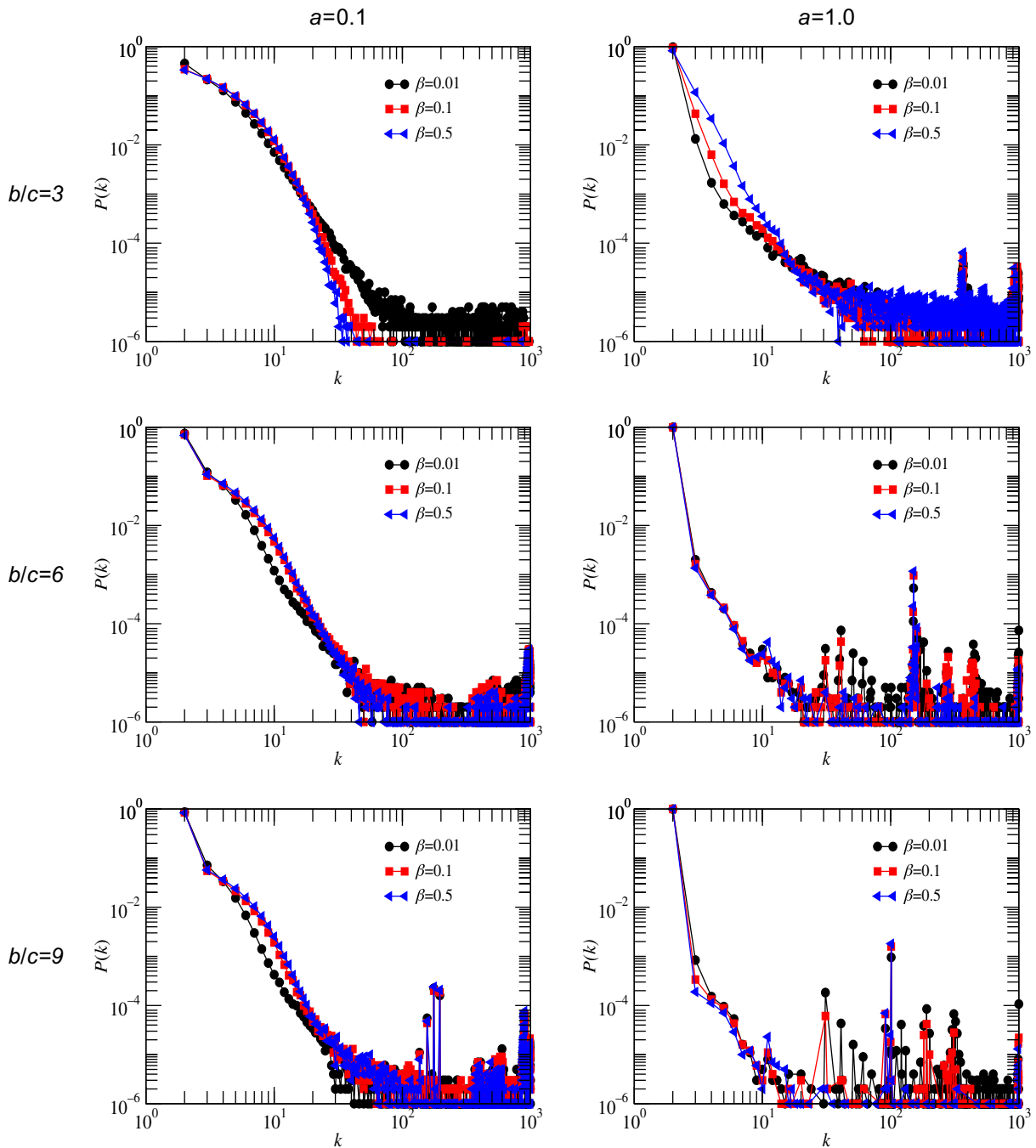
**Figure 2.** The average level of cooperation under strong selection ( $\beta = 1$ ) and  $\alpha = 1$  depending on the timescale of attachment,  $\tau$ . Cooperation benefits most from small values of  $\tau$ , i.e. when many new nodes are added before players update their strategies. For random attachment ( $\alpha = 0$ , inset) cooperation does not emerge, only for high benefit to cost ratios a few cooperators prevail ( $m = 2$ ,  $N_0 = 3$ , values obtained from  $10^2$  averages over networks of final size  $N = 1000$ , averaged when the network stops growing).

Typically, one is interested in the promotion of cooperation on different network structures. Figure 2 shows the average level of cooperation for strong selection as a function of  $\tau$ . It turns out that payoff preferential attachment increases the level of cooperation significantly compared to random attachment. This effect is also present for weak selection, but less pronounced. Cooperation increases most for small  $\tau$ , i.e. when many nodes are added before strategies are changed. This puts the system further from equilibrium, whereas the case of large  $\tau$  means that strategies have been equilibrated at least locally before the next new individual with a random strategy is added to the system. Note that for  $\tau$  larger than a certain value, cooperation levels become independent of  $\tau$ , which points out that playing once a given number of new players are incorporated is enough to reach a dynamical equilibrium.

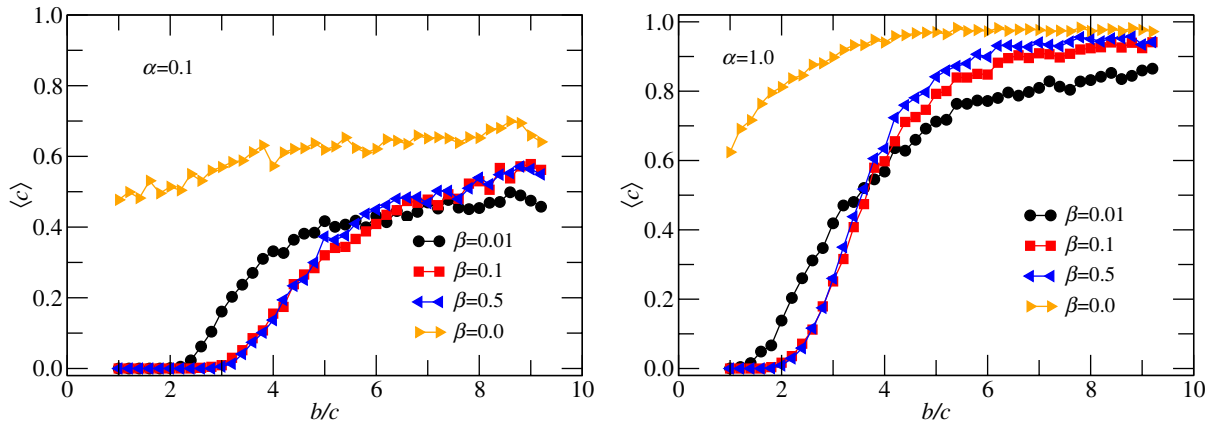
Since there is an interaction between strategy dynamics and network growth, the topology will change under selection. In figure 3, we show how the topology for the prisoner's dilemma changes with the benefit to cost ratio  $b/c$ , the intensity of selection  $\beta$  and the attachment parameter  $\alpha$  (see also figure 1). It turns out that the influence of the game on the degree distribution is relatively weak, for small degrees a clear difference is only found for large  $\alpha$  and small  $b/c$ . The distribution of the relatively few nodes with many connections, however, is more sensitive to changing either  $b/c$  or  $\beta$ .

#### 4. Promotion of cooperation in growing networks

As in most structured populations, cooperators that are disadvantageous in the prisoner's dilemma benefit from the spatial structure in a well-mixed population. Of course, this effect



**Figure 3.** Impact of the game dynamics on the degree distribution at the end of network growth. Left column corresponds to  $\alpha = 0.1$ , while the right one is for  $\alpha = 1$ . In general, game dynamics has only a weak impact on the topology of the system. However, there is a trend that stronger selection increases the number of nodes with fewer links and decreases the number of highly connected nodes ( $N_0 = 3$ ,  $m = 2$ ,  $\tau = 0.1$ , distributions averaged  $10^3$  over realizations of networks of  $10^3$  nodes each).



**Figure 4.** The average level of cooperation  $\langle c \rangle$   $10^4$  time steps after the network stops growing. For  $\alpha = 0.1$  (left) the level of cooperation exceeds 50% only for very high benefit to cost ratios  $b/c$ . For  $\alpha = 1.0$  (right), the abundance of cooperators is significantly higher. Even for neutral strategy dynamics ( $\beta = 0$ ), payoff preferential attachment can lead to high levels of cooperation in this case ( $N_0 = 3$ ,  $m = 2$ ,  $\tau = 0.1$ , averages over  $10^3$  different networks of size  $10^3$ ).

is larger when cooperation becomes more profitable, i.e. when the benefit to cost ratio  $b/c$  increases. It turns out that for weak payoff preferential attachment (small  $\alpha$ ), the promotion of cooperation is relatively weak and levels of cooperation beyond 50% are only reached when cooperation is very profitable, see figure 4. However, when the probability to attach to the most successful nodes becomes large (large  $\alpha$ ), then the average fraction of cooperators becomes larger, approaching one when the benefit cost ratio  $b/c$  is high.

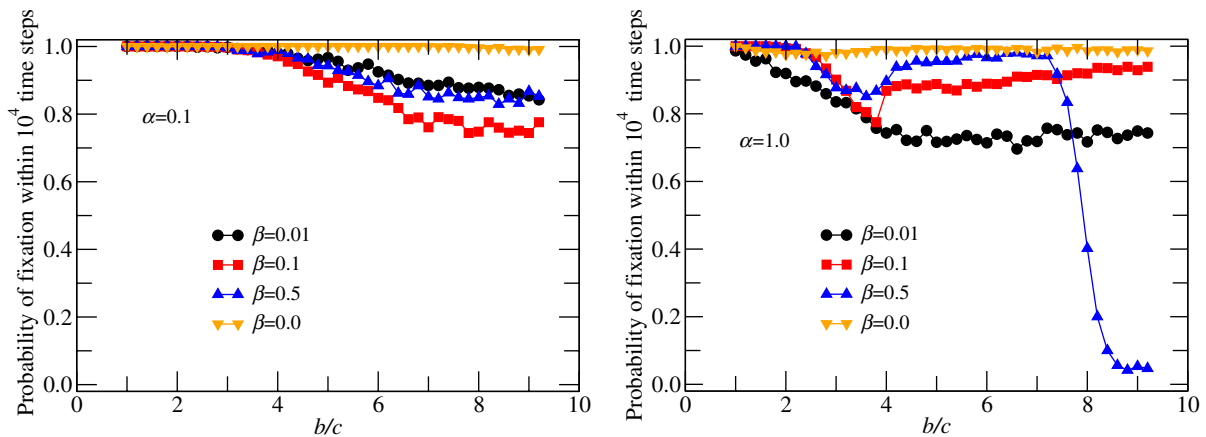
Interestingly, for small  $b/c$  ratios, the abundance of cooperators decreases with increasing  $\beta$ , whereas it increases with the intensity of selection for large  $b/c$  ratios. The existence of a threshold for intermediate  $b/c$  can be illustrated as follows for large  $\alpha$ : assume that we start from  $N_0$  fully connected cooperator nodes. For  $\tau < 1$ , we add  $1/\tau$  nodes with  $m = 2$  links, on average half of them defectors and half of them cooperators. All new players interact only with the initial cooperator nodes, such that an initial cooperator will on average obtain  $(m/N_0\tau)$  new links. The payoff of a new defector is  $mb$ . The average payoff of an initial cooperator is  $(b - c)(N_0 - 1 + (1/2)(m/N_0\tau)) - c(1/2)(m/N_0\tau)$ . Both payoffs are identical for

$$\frac{b}{c} = \frac{(1/\tau) + (N_0(N_0 - 1)/m)}{(1/2\tau) - N_0 + (N_0(N_0 - 1)/m)}. \quad (6)$$

For large  $b/c$ , cooperators will dominate in the very beginning of network growth. The threshold increases with  $\tau$  and decreases with  $N_0$ : the larger the initial cooperator cluster and the more nodes are added before strategies are updated, the easier it is for cooperation to spread initially. This argument shows qualitatively that a crossover in the abundance of cooperators should exist, and therefore that above a certain threshold, it is easier for cooperation to spread. This argument will hold quantitatively only in the very beginning of network growth.

In general, the average level of cooperation can be based on two very different scenarios: either it is the fraction of realizations of the process that ultimately ends in full cooperation, or it is the average abundance of cooperators in a network in which both cooperators and defectors





**Figure 5.** The dependence of the probability of fixation for one strategy within  $10^4$  time steps after growth has stopped on the attachment parameter  $\alpha$  (left column  $\alpha = 0.1$ , right column  $\alpha = 1$ ) for different intensities of selection  $\beta$ . For small  $\alpha$ , the degree distribution decays exponentially and fixation is relatively fast, regardless of the intensity of selection. For  $\alpha = 1.0$ , the network is more heterogeneous. As discussed in the text, for intermediate values of  $b/c$  ( $\approx 3.5$ ), the probability of fixation within  $10^4$  time steps is smaller than for higher and smaller  $b/c$ . For very high  $b/c$  and strong selection, one observes a coexistence of cooperators and defectors for a very long time rather than fixation for one of the strategies ( $N_0 = 3$ ,  $m = 2$ ,  $\tau = 0.1$ , averages over  $10^3$  independent realizations of a network of  $10^3$  nodes).

are present. For any finite intensity of selection  $\beta$ , we have  $T_{j \rightarrow i} > 0$ , regardless of the payoffs. Thus, after growth has stopped, our dynamics describes a recurrent Markov chain with two absorbing states in which all players follow one of the two strategies. Therefore, ultimately one of the two strategies will go extinct, in contrast to evolutionary processes that do not allow disadvantageous strategies to spread [25]. However, the time to extinction can become very large, in particular when the intensity of selection is high or the population size is large [27, 36]. In figure 5, we analyze this issue numerically. We address the probability that fixation (for either cooperation or defection) occurs within  $10^4$  time steps after the network has stopped growing. For small  $\alpha$ , the results follow the intuition from well-mixed populations: fixation within this time is more likely if the intensity of selection is weaker. With increasing benefit to cost ratio, fixation times increase and a fixation within the first  $10^4$  time steps becomes less and less likely.

For large  $\alpha$ , however, fixation is faster for strong selection (large  $\beta$ ) for a wide range of parameters. Only when the  $b/c$  ratio is very high are fixation times very large under strong selection. This is based on the peculiar structure of the network obtained for large  $\alpha$ . In addition, we observe an area in figure 5 where the fixation time increases slightly before it decreases again, i.e. the probability for fixation in the first  $10^4$  time steps has a minimum. Interestingly, this occurs for the range of  $b/c$  ratios where the average levels of cooperation intersect at 50% for the different intensities of selection. In this parameter region, neither cooperators nor defectors are clearly favored. Thus, they will initially both spread. When the abundance of both strategies is approximately constant in the beginning, then it will be more difficult to completely wipe

out one strategy later. Thus, the increased time of fixation in the parameter region where the abundance of cooperation becomes 50% makes intuitive sense.

### 5. Does cooperation benefit from growth or only from topology?

Typically, the promotion of cooperation in the prisoner's dilemma is analyzed on static networks. Our model allows a feedback between the game dynamics and the growth of the network.

What happens when the network stops growing? Typically, one would expect that defectors profit from growth, because there is a steady flow of new cooperators that they can potentially exploit. Thus, cooperation should increase if the game dynamics proceeds on the fully grown, static network. This has also been observed in a previous paper [25]. In contrast to that paper, here we have changed the game dynamics in such a way that individuals sometimes can also adopt a worse strategy. It has been shown that this seemingly small change can significantly decrease the level of cooperation [37]. The overall level of cooperation drops significantly and is only higher than 50% if cooperation is very profitable. In addition, the level of cooperation now decays once the network no longer grows, see figure 6. This means that cooperators, not defectors, benefit from the continuous supply of new players.

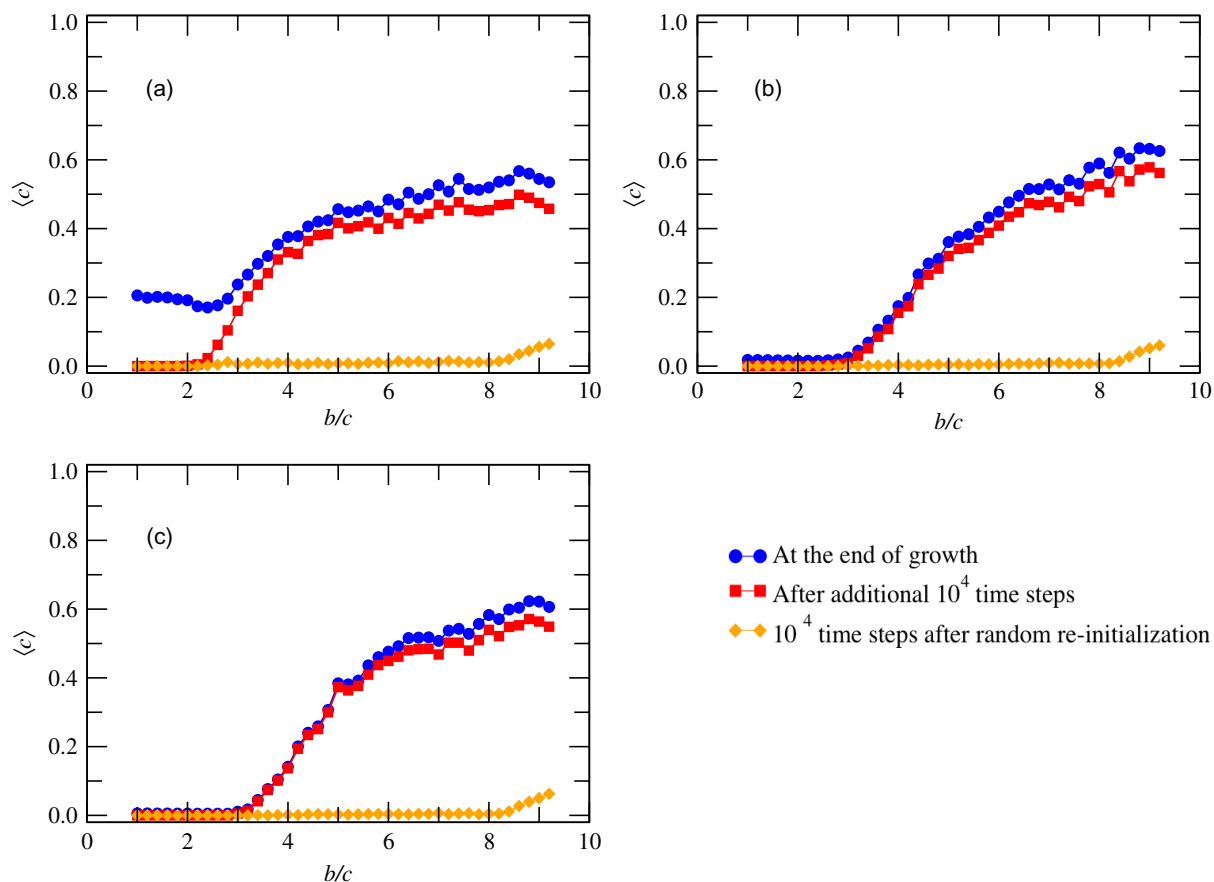
Next, we can ask whether the topologies that are obtained from the network growth are powerful promoters of cooperation at all. This can be tested by taking the fully grown, static network and run the game dynamics on the fixed network with initially random strategies, 50% cooperators and 50% defectors. Interestingly, this does not lead to any significant levels of cooperation (cf figure 6). Thus, our model of network growth based on payoff preferential attachment itself leads to comparably high levels of cooperation, while the resulting topology alone does not support cooperation in the prisoner's dilemma.

### 6. Discussion

Our model for evolutionary game dynamics in a growing, network-structured population is a dynamical network model [38]. Here, the network grows, in contrast to most models for evolutionary games on dynamical networks that consider a constant population size [39]–[51]. Individuals cannot break links and cannot control directly how many new individuals will establish connections with them.

An important difference with previous work [25] is that under strong payoff preferential attachment, the topology of the networks generated is dominated by the presence of a few hubs, which attract most of the links of the rest of the nodes. The existence of very few hubs and a large number of sparsely connected nodes in network models have been previously noticed [52]. In fact, it has been shown that when networks are grown following a nonlinear preferential attachment rule of the sort  $p_j = \frac{k_j^\nu}{\sum_{l=1}^N k_l^\nu}$ , with  $\nu > 1$ , star-like structures are obtained [53]. Here, we have shown that the same kind of network is produced when the dynamics driving the attachment process is dominated by the most successful players. Even when payoff preferential attachment is not too strong (for instance, for  $\alpha = 0.1$ ), super-hubs emerge, a clear mark that successful players are likely to attract many of the links of the new nodes.

If newcomers preferentially attach to the successful players in the game, then high levels of cooperation are possible. But this cooperation hinges upon the growth of the network,



**Figure 6.** Does cooperation benefit from the topology only or also from growth? Here we analyze the average level of cooperation in three cases: (i) once the network is fully grown, (ii) after the game dynamics has proceeded  $10^4$  additional steps beyond the growth phase of the network and (iii)  $10^4$  time steps after the fully grown network has been re-initialized with random strategies. Clearly, the grown network itself does not promote cooperation significantly. Instead, the growth phase is of crucial importance. The intensity of selection has only a minor influence on the phenomenon. (a)  $\beta = 0.01$ , (b)  $\beta = 0.1$  and (c)  $\beta = 0.5$  ( $N_0 = 3$ ,  $m = 2$ ,  $\tau = 0.1$ , averages over at least  $10^2$  networks of size  $10^3$ ,  $\alpha = 0.1$  in all cases).

the population structure alone would not lead to such high levels of cooperation. Thus, payoff preferential attachment differs from the usual promotion of cooperation in structured populations. In particular, it has been suspected that heterogeneous structures favor cooperative behavior due to the existence of hubs. However, as figure 6 shows, the presence of super-hubs is not enough to sustain cooperation in the networks grown following the scheme discussed here.

In other models, the probability of adopting a strategy that performs worse is zero [16, 23, 25]. In particular, together with synchronous updating of strategies, this can lead to evolutionary deadlocks, i.e. situations in which both strategies stably coexist. Here, we have adopted an update scheme in which individuals sometimes adopt a strategy that performs worse. Due to the presence of such irrational moves, sooner or later (often much later) one strategy

will reach fixation. However, when  $\beta$  and the ratio  $b/c$  are large enough, both cooperation and defection can coexist for a long time.

Let us also remark that our growth mechanism also has another interesting feature: it has been shown that the average level of cooperation obtained in static, scale-free networks, is robust to a wide range of initial conditions [54]. However, for the networks grown using the payoff preferential attachment, the initial average number of cooperators in the neighborhood of the super-hubs determines the fate of cooperation in the whole network, leading to a much more sensitive dependence on the initial state of the system. From this point of view, the weak dependence on the initial conditions reported in static scale-free networks is not trivial.

Finally, we point out that it would be of further interest to study the model discussed here with other  $2 \times 2$  games. As we have shown, the game dynamics seems to have a weak impact on the structure of the resulting networks. Whether or not this holds in general will elucidate the question of the influence of different games on the network formation process. For instance, within the model discussed in [45], different topologies emerge when different game dynamics are implemented.

In summary, our model shows that the interplay of game dynamics and network growth leads to complex network structures. Moreover, not only the structure of the interaction network is important for the evolution of cooperation, but also the particular way this structure is obtained. Our work shows that playing while growing can lead to radically different results with respect to the most studied cases in which game dynamics proceeds in static networks.

## Acknowledgments

We thank J M Pacheco for valuable comments. YM is grateful for the hospitality of the Max Planck Institute for Evolutionary Biology, where parts of this work were finished. We gratefully acknowledge funding by the COST action ‘Physics of Conflict and Cooperation’ (AT and YM), the Emmy-Noether Program of the DFG (AT), the MICINN (Spain) through Grants FIS2006-12781-C02-01 and FIS2008-01240. YM is supported by MICINN (Spain) through the Ramón y Cajal Program.

## References

- [1] von Neumann J and Morgenstern O 1944 *Theory of Games and Economic Behavior* (Princeton: Princeton University Press)
- [2] Maynard Smith J and Price G R 1973 The logic of animal conflict *Nature* **246** 15–8
- [3] Hofbauer J and Sigmund K 1998 *Evolutionary Games and Population Dynamics* (Cambridge: Cambridge University Press)
- [4] Nowak M A and Sigmund K 2004 Evolutionary dynamics of biological games *Science* **303** 793–9
- [5] Sato Y, Akiyama E and Farmer J D 2002 Chaos in learning a simple two-person game *Proc. Natl Acad. Sci. USA* **99** 4748–51
- [6] Sato Y and Crutchfield J P 2003 Coupled replicator equations for the dynamics of learning in multiagent systems *Phys. Rev. E* **67** 015206
- [7] Berg J and Engel A 1998 Matrix games, mixed strategies and statistical mechanics *Phys. Rev. Lett.* **81** 4999–5002
- [8] Vainstein M H and Arenzon J J 2001 Disordered environments in spatial games *Phys. Rev. E* **64** 051905
- [9] Galla T 2007 Two-population replicator dynamics and number of Nash equilibria in random matrix games *Europhys. Lett.* **78** 20005

- [10] Helbing D 1993 Stochastic and Boltzmann-like models for behavioral changes, and their relation to game theory *Physica A* **193** 241–58
- [11] Traulsen A, Claussen J C and Hauert C 2006 Coevolutionary dynamics in large, but finite populations *Phys. Rev. E* **74** 011901
- [12] Lindgren K and Nordahl M G 1994 Evolutionary dynamics of spatial games *Physica D* **75** 292–309
- [13] Szabó G and Tóke C 1998 Evolutionary prisoner's dilemma game on a square lattice *Phys. Rev. E* **58** 69
- [14] Abramson G and Kuperman M 2001 Social games in a social network *Phys. Rev. E* **63** 030901
- [15] Schweitzer F, Behera L and Mühlenbein H 2002 Evolution of cooperation in a spatial prisoner's dilemma *Adv. Complex Syst.* **5** 269–99
- [16] Santos F C and Pacheco J M 2005 Scale-free networks provide a unifying framework for the emergence of cooperation *Phys. Rev. Lett.* **95** 098104
- [17] Perc M 2006 Coherence resonance in a spatial prisoner's dilemma game *New J. Phys.* **8** 22
- [18] Perc M, Szolnoki A and Szabó G 2008 Restricted connections among distinguished players support cooperation *Phys. Rev. E* **78** 066101
- [19] Boccaletti S, Latora V, Moreno Y, Chávez M and Hwang D U 2006 Complex networks: structure and dynamics *Phys. Rep.* **424** 175–308
- [20] Hauert C 2002 Effects of space in  $2 \times 2$  games *Int. J. Bifurcation Chaos Appl. Sci. Eng.* **12** 1531–48
- [21] Ohtsuki H, Hauert C, Lieberman E and Nowak M A 2006 A simple rule for the evolution of cooperation on graphs *Nature* **441** 502–5
- [22] Szabó G and Fáth G 2007 Evolutionary games on graphs *Phys. Rep.* **446** 97–216
- [23] Floría L M, Gracia-Lázaro C, Gómez-Gardeñes J and Moreno Y 2009 Social network reciprocity as a phase transition in evolutionary cooperation *Phys. Rev. E* **79** 026106
- [24] Roca C P, Cuesta J A and Sánchez A 2009 On the effect of spatial structure on the evolution of cooperation arXiv:0806.1649
- [25] Poncela J, Gómez-Gardeñes J, Floría L M, Sánchez A and Moreno Y 2008 Complex cooperative networks from evolutionary preferential attachment *PLoS One* **3** e2449
- [26] Blume L E 1993 The statistical mechanics of strategic interaction *Games Econ. Behav.* **5** 387–424
- [27] Traulsen A, Pacheco J M and Nowak M A 2007 Pairwise comparison and selection temperature in evolutionary game dynamics *J. Theor. Biol.* **246** 522–9
- [28] Santos F C and Pacheco J M 2006 A new route to the evolution of cooperation *J. Evol. Biol.* **19** 726–33
- [29] Pusch A, Weber S and Porto M 2008 Impact of topology on the dynamical organization of cooperation in the prisoner's dilemma game *Phys. Rev. E* **77** 036120
- [30] Barabási A L, Albert R and Jeong H 1999 Mean-field theory for scale-free random networks *Physica A* **272** 173–87
- [31] Dorogovtsev S N, Mendes J F and Samukhin A N 2000 Structure of growing networks with preferential linking *Phys. Rev. Lett.* **85** 4633–6
- [32] Rapoport A and Chammah A M 1965 *Prisoner's Dilemma* (Ann Arbor: University of Michigan Press)
- [33] Axelrod R 1984 *The Evolution of Cooperation* (New York: Basic Books)
- [34] Nowak M A 2006 Five rules for the evolution of cooperation *Science* **314** 1560–3
- [35] Nowak M A and May R M 1992 Evolutionary games and spatial chaos *Nature* **359** 826–9
- [36] Antal T and Scheuring I 2006 Fixation of strategies for an evolutionary game in finite populations *Bull. Math. Biol.* **68** 1923–44
- [37] Ohtsuki H and Nowak M A 2006 The replicator equation on graphs *J. Theor. Biol.* **243** 86–97
- [38] Gross T and Blasius B 2008 Adaptive coevolutionary networks—a review *J. R. Soc. Interface* **5** 259–71
- [39] Skyrms B and Pemantle R 2000 A dynamical model of social network formation *Proc. Natl Acad. Sci. USA* **97** 9340–6
- [40] Ebel H and Bornholdt S 2002 Coevolutionary games on networks *Phys. Rev. E* **66** 056118
- [41] Ebel H and Bornholdt S 2002 Evolutionary games and the emergence of complex networks arXiv:cond-mat/0211666

- [42] Zimmermann M G, Eguíluz V M and San Miguel M 2005 Cooperation and emergence of role differentiation in the dynamics of social networks *Am. J. Sociol.* **110** 977
- [43] Pacheco J M, Traulsen A and Nowak M A 2006 Active linking in evolutionary games *J. Theor. Biol.* **243** 437–43
- [44] Pacheco J M, Traulsen A and Nowak M A 2006 Co-evolution of strategy and structure in complex networks with dynamical linking *Phys. Rev. Lett.* **97** 258103
- [45] Santos F C, Pacheco J M and Lenaerts T 2006 Cooperation prevails when individuals adjust their social ties *PLoS Comput. Biol.* **2** 1284–91
- [46] Biely C, Dragosit K and Thurner S 2007 Prisoners dilemma on dynamic networks under perfect rationality *Physica D* **228** 40–8
- [47] Pacheco J M, Traulsen A, Ohtsuki H and Nowak M A 2008 Repeated games and direct reciprocity under active linking *J. Theor. Biol.* **250** 723–31
- [48] Van Segbroeck S, Santos F C, Nowe A, Pacheco J M and Lenaerts T 2008 The evolution of prompt reaction to adverse ties *BMC Evol. Biol.* **8** 287
- [49] Van Segbroeck S, Santos F C, Lenaerts T and Pacheco J M 2009 Reacting differently to adverse ties promotes cooperation in social networks *Phys. Rev. Lett.* **102** 058105
- [50] Gómez-Gardeñes J, Campillo M, Floría L M and Moreno Y 2007 Dynamical organization of cooperation in complex topologies *Phys. Rev. Lett.* **98** 108103
- [51] Gómez-Gardeñes J, Poncela J, Mario Floría L and Moreno Y 2008 Natural selection of cooperation and degree hierarchy in heterogeneous populations *J. Theor. Biol.* **253** 296–301
- [52] Leyvraz F and Redner S 2002 Scaling theory for migration-driven aggregate growth *Phys. Rev. Lett.* **88** 068301
- [53] Krapivsky P L, Redner S and Leyvraz F 2000 Connectivity of growing random networks *Phys. Rev. Lett.* **85** 4629–32
- [54] Poncela J, Gómez-Gardeñes J, Floría L M and Moreno Y 2007 Robustness of cooperation in the evolutionary prisoner's dilemma on complex networks *New J. Phys.* **9** 184